Name ______________________

Astronomy H161 – An Introduction to Solar System Astronomy
Winter Quarter 2009 – Prof. Gaudi
Homework #5

Due Monday, February 23 in class

No late homework will be accepted.

---

1) About 15 years ago, then NASA Administrator Dan Goldin put forward a bold vision for the next half century to take pictures of a "pale blue dot", an Earth-like planet around another star. The star might be 10 parsecs away. (1 parsec = 3×10^{16} meters. It is the distance at which an object that is 1 AU across subtends 1 arcsecond.) It is not enough just to detect light from the planet: Goldin wanted to resolve the surface of the planet into a 25 × 25 array of "picture elements" (or "pixels" for short).
   a) How big is each pixel in meters?
   b) If the planet is at 10 parsecs, how big is each pixel in radians? How big in arcseconds? (Remember 57 degrees in a radian, 60 arcminute in a degree, 60 arcseconds in an arcminute.)
   c) Explain why this project has to be done from space.
   d) Since the object is to image a "pale blue dot", one will want to see it in blue light ($\lambda = 0.4\mu m$). Based on this wavelength and your answer to part (b), calculate how big will the telescope mirror system have to be.
   e) Can a single mirror be built this big? If not, what sort of mirror system could be used?
2) Radioactive dating of rocks is crucial to our understanding of the history of the Earth and Moon, which in turn are the key to dating the evolution of the entire solar system. At first sight, however, radioactive dating seems to depend on a series of unsupportable assumptions. For example, one often hears that uranium decays into lead at a known rate. So one can just look in a rock, see how much uranium and lead it has, and get the age from the amount of time it takes for decay of uranium to

produce that much lead. The trouble is, how does one know that the rock didn't start out with a lot of lead to begin with? This exercise is aimed at helping you understand how radioactive dating avoids this apparent pitfall. It uses the example of Rubidium (Rb^{87}) decaying into Strontium (Sr^{87}). When a rock congeals out of a molten mass, it will already have some amount of Rb^{87} and some Sr^{87}, It will also have some Sr^{86}, another isotope of Strontium, which is chemically identical to Sr^{87}, but which does not decay and is not a decay product of any other element. Thus, the amounts of Rb^{87} and Sr^{87} change with time, but the amount Sr^{86} stays the same. As the rock congeals, different minerals form. These minerals have different ratios of Rubidium to Strontium (depending on their chemistry), but the ratios of Sr^{87}/Sr^{86} must be the same because these isotopes are chemically identical.

a) The attached table has 6 mineral samples, 3 from one rock (1A, B, C) and 3 from another (2A, B, C). Fill in the ratios Sr^{87}/Sr^{86} and Rb^{87}/Sr^{86} at time "0 Gyr" (i.e. when the rock first formed). Verify that for each rock, the 3 ratios Sr^{87}/Sr^{86} are the same.

b) Every billion years, 1.46% of the Rb^{87} decays into Sr^{87}. Thus, after 1, 2, 3, 4, and 5 Gyr, the fraction of the original Rubidium that remains is **0.9854**, **0.9709**, **0.9567**, **0.9427**, and **0.9289**, respectively. For each of the six minerals and for each time elapsed, calculate the amount of Rubidium remaining and enter it in the table.

c) Since this Rubidium has become Sr^{87}, add the amount of "decayed" Rubidium to the Sr^{87} for each mineral and each time.

d) The amount of Sr^{86} is unchanged. Enter these constant values and then compute the ratios Sr^{87}/Sr^{86} and Rb^{87}/Sr^{86}.

e) Plot Sr^{87}/Sr^{86} (vertical axis) versus Rb^{87}/Sr^{86} (horizontal axis) for all points of the three minerals in rock 1. Now for each of the five elapsed times (1, 2, 3, 4 and 5 Gyr), connect the three points from the three minerals. These should form straight lines. If they do not, either you or I have done something wrong. What do these five lines have in common? What is different about them?

f) Repeat this procedure on a separate plot for the points of the three minerals in rock 2. What is similar about the 5 Gyr line in rock 2 to the 5 Gyr line in rock 1? What is different? What is similar about the two 4 Gyr lines, the two 3 Gyr lines, 2 Gyr lines, and 1 Gyr lines?

g) So far, the rocks have been just theoretical. That is, you pretended you knew the initial composition of the rocks and calculated how that composition changed. Now you are given a real rock and you measure the composition of three minerals.

3A) Sr^{86}: 46.75; Sr^{87}: 35.46; Rb^{87}: 6.14

3B) Sr^{86}: 64.82; Sr^{87}: 50.61; Rb^{87}: 30.42

3C) Sr^{86}: 90.38; Sr^{87}: 72.90; Rb^{87}: 78.04

How old is the rock? Give your answer to the nearest 0.1 Gyr.

h) What was the original ratio of Sr^{87}/Sr^{86} when the rock was first formed?