Astronomy 2291 HW 2

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$\cancel{1}, \cancel{2}, \cancel{3}, \cancel{4}, \cancel{5}, \cancel{6}, \cancel{7}, \cancel{8}.$

1. Kepler III: $\boxed{P^2 = ka^3}$

In this case, $a = R_E + h$

$$P_{LEO} = k^{1/2} a^{3/2} = k^{1/2} a^{3/2}$$
$$= k^{1/2}(R_E+h)^{3/2}$$
$$= k^{1/2}\left(R_E\left(1+\frac{h}{R_E}\right)\right)^{3/2}$$
$$= \underbrace{k^{1/2} R_E^{3/2}}\left(1+\frac{h}{R_E}\right)^{3/2}$$
$$= C\left(1+\frac{h}{R_E}\right)^{3/2} \quad \text{where } C = \left(kR_E^3\right)^{1/2}$$

Since $h \ll R_E$,
$$\frac{h}{R_E} \ll 1.$$

Binomial approximation for $(1+\varepsilon)^x$ when ε is small,
$$(1+\varepsilon)^x \simeq 1 + x\varepsilon$$

So, $P_{LEO} = C\left(1+\frac{h}{R_E}\right)^{3/2} \simeq \boxed{C\left(1+\frac{3}{2}\frac{h}{R_E}\right)}$

$$C = \left(\frac{4\pi^2 R_E^3}{G M_E}\right)^{1/2}$$

Plug in numbers to get $\boxed{C = 84.4 \text{ min}}$

2. P = 1 sidereal day = 23.93 hr = 86148 s

a. $a = \left(\frac{G M_\oplus P^2}{4\pi^2}\right)^{1/3} \longrightarrow a_{gs} = \boxed{4.22 \times 10^9 \text{ cm}}$

b. $v = d/t \quad d = 2\pi a_{gs} \quad t = 1 \text{ day} = 86148 \text{ s} \Rightarrow v_{gs} = \boxed{3.08 \times 10^5 \text{ cm s}^{-1}}$

3. $v_{LEO} = 2\pi a_{LEO} / P_{LEO}$

a. From (1), $P_{LEO} = C\left(1+\frac{3}{2}\frac{h}{R_E}\right)$

Plug in C, R_E. Given $h = 300$ km

$\boxed{P_{LEO} = 5427 \text{ s}} \qquad 2\pi a_{LEO} = \boxed{6.678 \times 10^8 \text{ cm}}$

$v_{LEO} = \boxed{7.73 \times 10^5 \text{ cm s}^{-1}}$

b. $a_{to} = \frac{1}{2}(a_{LEO} + a_{gs}) = 2.44 \times 10^9$ cm $\qquad P_{to} = \left(\frac{4\pi^2}{GM_\oplus}\right)^{1/2} a_{to}^{3/2} = 3.79 \times 10^4$ s

$v(r) = \frac{2\pi a_{to}}{P_{to}}\left(\frac{2a_{to}}{r} - 1\right)^{1/2} \qquad v_{pe} = v(r_{LEO}) = \boxed{1.02 \times 10^6 \text{ cm s}^{-1}}$

$v_{boost} = v_{pe} - v_{LEO} = \boxed{2.47 \times 10^5 \text{ cm s}^{-1}}$

c. $v_{ap} = v(r_{gs}) = \boxed{1.60 \times 10^5 \text{ cm s}^{-1}} \qquad v_{insert} = v_{gs} - v_{ap} = \boxed{1.48 \times 10^5 \text{ cm s}^{-1}}$

d. $t = P_{to}/2 = 1.895 \times 10^4$ s $\Rightarrow t = \boxed{5.26 \text{ hr}}$

5. $K = \frac{1}{2} m_e v^2 \Rightarrow v = \left(\frac{2K}{m_e}\right)^{1/2}$

$K = 5.1 \text{ eV} \cdot \frac{1 \text{ erg}}{6.24 \times 10^{11} \text{ eV}} = 8.17 \times 10^{-12} \text{ erg}$
$\text{g cm}^2 \text{s}^{-2}$

$F^{v} = \left(\frac{1.63 \times 10^{-11} \text{ g cm}^2 \text{s}^{-2}}{9.1 \times 10^{-28} \text{ g}}\right)^{1/2} \Rightarrow v_e = \boxed{1.3 \times 10^8 \text{ cm s}^{-1}}$ a

or proton, use mass of proton.

$v = \left(\frac{2K}{m_p}\right)^{1/2} = \left(\frac{1.63 \times 10^{-11} \text{ g cm}^2 \text{s}^{-2}}{1.67 \times 10^{-24} \text{ g}}\right) \Rightarrow v_p = \boxed{3.1 \times 10^6 \text{ cm s}^{-1}}$ b

$\langle K \rangle = \frac{3}{2} k_B T \rightarrow T = \frac{2}{3} (8.617 \times 10^{-5} \text{ eV K}^{-1})^{-1} (5.1 \text{ eV}) \Rightarrow T = \boxed{3.95 \times 10^4 \text{ K}}$ c

7. $L = 100 \text{ W} = 100 \text{ J s}^{-1} = 10^9 \text{ erg s}^{-1}$
$T = 2900 \text{ K}$

$L = S \sigma_{SB} T^4 \Rightarrow S = L / \sigma_{SB} T^4 = 10^9 / (5.67 \times 10^{-5} \cdot 2900^4) = \boxed{0.25 \text{ cm}^2}$ a

$S = \pi D \ell \Rightarrow \ell = S / \pi D$ $\quad \boxed{\ell = 16 \text{ cm}}$ b

$\rho = 19.25 \text{ g/cc}$ (wikipedia)

$m = \rho V = \pi r^2 \ell \Rightarrow \boxed{m = 0.006 \text{ g}}$ c.

8. $\lambda_{peak} = \frac{2900 \, \mu\text{m K}}{T = 2900 \text{ K}} = \boxed{1 \, \mu\text{m}}$ a

Compute $B(\lambda)$ for $\lambda = 0.4, 0.7$ and $1 \, \mu\text{m}$

$B(0.4 \mu\text{m}) / B(1 \mu\text{m}) = \boxed{0.06}$ b

$B(0.7 \mu\text{m}) / B(1 \mu\text{m}) = \boxed{0.71}$

c. Most of the energy is emitted at wavelengths we cannot see. Not efficient.

4. $\rho_* = \frac{3 M_*}{4 \pi R_*^3} = \boxed{1.4 \text{ g/cc}}$ a $\qquad \rho_J = \frac{3 M_J}{4 \pi R_J^3} = \boxed{1.2 \text{ g/cc}}$ a

$R_R \approx 2.44 R_* \left(\frac{\rho_*}{\rho_J}\right)^{1/3} = 2.57 R_* = \boxed{0.012 \text{ AU}}$ b.

$P^2 = K a^3 \qquad K = 1 \text{ AU}^{-3} \text{yr}^2$

$a = 0.012 \text{ AU} \Rightarrow \boxed{P = 0.0013 \text{ yr} \simeq 11.4 \text{ hrs}}$ c.

$R_R = 2.57 R_* \Rightarrow$ $\boxed{\text{outside}}$ d.

6. Let f be fraction of light absorbed after passing through glass of thickness x.

$I(x) = I_0 e^{-n\sigma x} = I_0 e^{-x/\lambda} \qquad \lambda = \frac{1}{n\sigma}$

$\frac{I(x)}{I_0} = e^{-x/\lambda} \qquad \lambda$ is constant.

when $x = 0.5$ m $\quad I/I_0 = 0.5$

solve for $\lambda = 0.72$ m

$I/I_0 = 1-f$

$$\boxed{x = -\lambda \ln(1-f)}$$

Plug in f to get $x = \boxed{\begin{array}{l} 1.66 \text{ m} \\ 3.32 \text{ m} \\ 4.97 \text{ m} \end{array}}$ $\begin{array}{l} a. \\ b. \\ c. \end{array}$