

Problem Set #1
Due Monday, September 14 in class

Instructions

Please work out your answers on separate sheets of paper and then staple them together and hand them in. Write your name on the top of each sheet of paper, in case they get separated. Please show all steps in your calculations, including any sketches you might need. Be careful to properly evaluate and give units where needed, and also be careful with significant figures. Calculations given without “showing the work” will receive zero credit even if the final answer is correct.

While you may discuss these problems with your fellow students, you must work out the details and write up the answers yourself.

Welcome to the Zolar System!

Aliens abduct you and deposit you on a desolate, abandoned planet in another planetary system. They give you food, clothes, and shelter, as well as a precise atomic clock, which ticks off time in units of ‘zours’. They also give you a compass, a device that allows you to precisely measure positions of objects in the sky, and a device that allows you to measure distances in units of ‘zilometers’. Being quite cruel, they do not give you any method of communication with home, or any source of entertainment. They do give you rudimentary writing utensils and paper. Without anything more interesting to do, you are forced gaze at the sky and document and understand the motion of the heavenly bodies. Fortunately, having just learned about similar struggles in our class, you are eager to see if you can apply your knowledge to your newfound home.

Over the first few hundred zours, you observe stars that rise in the East, and set in the West, moving with uniform motion on perfect circles around a point that is 30.000 degrees above your Northern horizon. You also observe a resolved disk in the sky that rises due East, and sets due west. Getting the hint, you decide to name this object the Zun. After many cycles you determine that the Zun always rises due East and sets due West, and always reaches the same elevation in the sky of 60.000 degrees above the Southern horizon at its highest point. You also measure that the durations of daylight and darkness are always equal, and that the length of time between Zunrises is constant and equal to 10.101 zours, which you decide to call a zolar zay.

Not being very imaginative, you further decide to name the body upon which you are standing the Zearth, and the planetary system in which you have been deposited the Zolar System. You wisely decide to adopt a Zun-centric model for the Zolar System. Based on what you’ve learned so far:

1. What is your latitude on Zearth?
2. What is the obliquity of Zearth’s orbit with respect to its celestial equator? In other words, what is the tilt of Zearth’s axis with respect to the normal of its orbit around the Zun?
3. What is the path that the Zun takes on the sky?
4. What is the eccentricity of Zearth’s orbit about the Zun? How did you infer this?

You notice without surprise that the Zun is slowly drifting Eastward relative to the stars. Eventually you wait long enough, and you find that the Zun has returned back to the same spot with respect to the stars after a period of 1000.000 zours. You call this unit of time a zear.

5. What is the value of the sidereal zay for the Zearth in zours?

You notice another disk in the sky that is paler and smaller than that of the Zun. You notice it goes through phases, and so decide to call it the Zoon. You determine that the Zoon has an angular diameter of 1.1459 degrees, 10 times smaller than the angular diameter of the Zun. Recalling Aristarchus, you wait until the Zoon is exactly at quarter phase, and then measure the angle between the Zoon and the Zun to be 89.42704 degrees.

6. How much further is the Zun from the Zearth than the Zoon?

7. What is the radius of the Zun relative to the radius of the Zoon?

You then go on to measure the total diurnal parallax of the Zoon (i.e., the *total* shift of the position of the Zoon relative to the stars as seen from opposite ends of the Zearth) to be 11.4211 degrees.

8. What is the distance of the Zoon from the Zearth in units of the radius of the Zearth?

Recalling Eratosthenes, you decide to start walking due south. Eventually, after a very long walk of 523.599 zilometers, you reach a point where the Zun reaches the zenith in the middle of the zay.

9. What is the radius of the Zearth in zilometers?

10. What are the distance to, and radius of, the Zoon in zilometers?

11. What is the radius of the Zun in zilometers?

You define the ZAU as a unit of distance equal to the distance of the Zearth to the Zun.

12. What is 1 ZAU in zilometers?

13. What are the ratios of the radii of the Zearth to the Zoon and Zun to the Zearth?

You decide to try test your results by seeing if you can predict and measure the diurnal parallax of the Zun.

14. What is the diurnal parallax of the Zun? Do you think you should be able to measure this?

After some additional observing, you notice that two of the stars actually move relative to the other stars, and so you suspect that they are additional planets of the Zun. One of the planets, which you decide to call Zenus, is clearly an inferior planet: it never gets further than 30.000 degrees from the Zun. You measure a synodic period for this planet of 546.92 zours.

15. What is the sidereal period of Zenus in zours? In sidereal zays? In zears?

16. Using only geometry, what is the distance of Zenus from the Zun in ZAU? In zilometers?

The other planet, which you decide to call Zupiter, is clearly a superior planet. You measure a synodic period of 1032.655 zours for Zupiter. You also determine that the time between opposition and eastern quadrature for Zupiter is 24.17011 days.

17. What is the sidereal period of Zupiter in zours? In sidereal zays? In zears?

18. Using only geometry, what is the distance of Zupiter from the Zun in ZAU? In zilometers?

19. Sketch orbits of the Zearth, the Zoon, Zenus, and Zupiter around the Zun to scale as seen from above.

Using the orbital distances and the periods of Zearth, Zenus, and Zupiter that you have computed:

20. Determine whether or not the orbits of objects in the Zolar system obey Kepler's third law, $P^2 = Ka^3$.

21. If so, what is the constant of proportionality K in units of ZAU and zears? In units of sidereal zays and ZAU? In units of zours and zilometers?

You notice a comet in the sky, and after tracking its position very carefully, you are able to determine that it reached a perizelion distance of 0.1 ZAU, and its orbit has an eccentricity of 0.999.

22. What is its semi-major axis in ZAU? Draw the inner part of the comet's orbit on your diagram from part (19).

23. What is its period in zears?

Having performed admirably, the aliens return you back home, where you are immediately obtain celebrity status.