

Problem Set #2
Due Monday, October 19 in class

Instructions

Please work out your answers on separate sheets of paper and staple them together to hand in. Write your name on the top of each sheet of paper, in case they get separated. Please show all steps in your calculations, including any sketches you might need. Be careful to properly evaluate and give units where needed, and also be careful with significant figures. Calculations given without “showing the work” will receive zero credit even if the final answer is correct.

While you may discuss these problems with other students, you must work out the details and write up the answers yourself.

1. A “Low-Earth Orbit” (LEO) is one in which the height of the satellite above the surface of the Earth, h , is much smaller than the radius of the Earth ($h \ll R_E$). Show that in the limit $h \ll R_E$, the orbital period of a satellite in a circular LEO is given approximately by the formula:

$$P_{LEO} = C \left(1 + \frac{3h}{2R_E} \right)$$

where C is a constant. Compute the value of C in units of minutes.

2. Earth-orbiting communications and weather satellites are often put into geosynchronous orbits with a period of $P_{orbit} = 1$ sidereal day.
 - a. What is the semimajor axis, a_{gs} , of a geosynchronous orbit?
 - b. What is the orbital velocity, v_{gs} , of a satellite on a circular geosynchronous orbit?
3. A satellite is delivered into a circular Low-Earth Orbit (LEO) with height $h=300$ km above the Earth’s surface. A booster rocket attached to the satellite is then used to lift the satellite into a geosynchronous circular orbit via the transfer orbit we derived in class.
 - a. What is the satellite’s orbital velocity, v_{LEO} , while in its initial circular Low-Earth Orbit?
 - b. What is the orbital velocity at pericenter, v_{pe} , of the Hohmann transfer orbit between LEO and geosynchronous orbit, and the subsequent $\Delta v_{boost} = v_{pe} - v_{LEO}$ boost needed?
 - c. What is the orbital velocity at apocenter, v_{ap} , of the Hohmann transfer orbit, and the $\Delta v_{insert} = v_{gs} - v_{ap}$ insertion burn needed to get it onto the geosynchronous orbit?
 - d. How long does it take in hours for the satellite to travel from its initial circular LEO to its final circular geosynchronous orbit?
4. A surprising result of searches for planets around other stars is the number of “Hot Jupiters”: Jupiter-mass planets found to be orbiting extremely close to their parent stars. This raises the question of how close is too close for a Jupiter-like planet. Consider a Sun-like star ($M_S = 1.9891 \times 10^{30}$ kg and $R_S = 6.955 \times 10^5$ km), orbited by a single Jupiter-like planet ($M_J = 1.90 \times 10^{27}$ kg and $R_J = 71492$ km).
 - a. What is the mean density of the star and the hot Jupiter, respectively?
 - b. How close can the hot Jupiter get to its host star before it is tidally disrupted? Express your answer in AU and Solar Radii (R_S).
 - c. What is the orbital period in hours of a hot Jupiter orbiting at the tidal disruption distance?
 - d. Is this tidal disruption distance inside or outside the host star itself?
5. A neutral sodium atom requires an energy of 5.1eV to ionize it.

- a. What is the speed of a free electron that has the same kinetic energy as the ionization energy of neutral atomic sodium?
 - b. What is the speed of a free proton that has the same kinetic energy as the ionization energy of neutral atomic sodium?
 - c. What is the temperature, T , in Kelvin of a gas of free electrons whose average electron kinetic energy will be the same as the ionization energy of neutral atomic sodium?
6. A slab of glass 0.5-meters thick absorbs 50% of the light passing through it. Compute how thick of a slab of the identical glass you would need that...
- a. absorbs 90% of the light passing through it?
 - b. absorbs 99% of the light passing through it?
 - c. absorbs 99.9% of the light passing through it?
7. An incandescent light bulb (a dying breed!) uses a hot filament to make light. Consider such a light bulb with a luminosity of $L=100$ Watts and a filament temperature of $T=2900\text{K}$.
- a. What is the total surface area of the filament?
 - b. If the filament is a cylindrical wire with a diameter of $50\mu\text{m}$ (5×10^{-5} meters), how long would the wire in this bulb need to be?
 - c. Since 1910, light bulb filaments have been made of Tungsten wire. What mass of tungsten would you need to make a 100 Watt light bulb filament? You will need to look up the density of Tungsten (remember to state your source of information).
8. Wien's Law states that the peak wavelength of a blackbody function occurs at a wavelength of

$$\lambda_{peak} \approx \frac{2900 \mu\text{m K}}{T}$$

where T is the temperature in Kelvin and λ_{peak} is the peak wavelength in μm .

- a. What is the peak wavelength of a conventional incandescent light bulb with a filament temperature of 2900K?
- b. The human eye can see wavelengths between 0.4 and $0.7\mu\text{m}$. Using the blackbody intensity formula

$$B_{\lambda} = \pi I_{\lambda} = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

compare the blackbody intensity for a $T=2900\text{K}$ filament at 0.4 and $0.7\mu\text{m}$ to the intensity at λ_{peak} , you computed in part a.

- c. What does your answer imply about the energy efficiency of incandescent light bulbs?