Astronomy 2291
Autumn Semester 2015

# Problem Set #3

Due Monday, November 16 in class

## Instructions

Please work out your answers on separate sheets of paper and then staple them together and hand them in. In cases where a calculation is called for, please show all steps in the calculation, including any sketches you might need, and be careful to properly evaluate units and significant figures. Calculations given without "showing the work" will receive zero credit even if the final answer is correct.

While you may discuss these problems with your fellow students on these problems, you must work out the details and write up the answers yourself.

1. The decay of radioactive isotopes in the Earth's interior results in a mean heat flux through the Earth's surface of about $5\times10^{-2}$ W m$^{-2}$.
    a. Express the heat flux from radioactive decay as a fraction of the total thermal flux due to re-radiated solar energy emitted by the Earth (assume a surface temperature of T=290K).
    b. If the Sun suddenly went dark and radioactive decay became the *only* source of heat on the Earth, what would be the Earth's new equilibrium surface temperature?
2. Titan is the largest moon in the Saturn system, notable for its heavy $N_2$ atmosphere and liquid methane lakes.
    a. What is the mass and radius of Titan? Give the source of your data.
    b. Compute the mean mass density, $\bar{\rho}$, of Titan.
    c. What does this mean density suggest about the likely composition of Titan? Explain why.
    d. The exobase of Titan is about 1500 km above its surface. Compute the escape velocity at the exobase of Titan.
    e. Compute the thermal (RMS) velocity at the exobase of Titan for Nitrogen, assuming an exobase temperature of 90K.
    f. What is the ratio of the escape velocity to the thermal velocity for Nitrogen at the exobase of Titan?
3. Asteroid 2007 $EB_{26}$ has an elliptical orbit with a=0.574 AU and e=0.787. Assume that it is slowly rotating with no atmosphere and has an albedo of A≈0.05.
    a. What is the temperature of the subsolar point on 2007 $EB_{26}$ at aphelion?
    b. What is the temperature of the subsolar point on 2007 $EB_{26}$ at perihelion?
4. Sedna, a large Trans-Neptunian Object, has an albedo of A≈0.32. It has a highly elliptical orbit (e=0.8549) with a semi-major axis of a=524 AU. Its mass is estimated to be ~$10^{21}$ kg, and its radius is ~500 km.
    a. What is Sedna's rotation period? Provide the original reference. Is Sedna a fast or slow rotator?
    b. What is Sedna's equilibrium temperature at perihelion?
    c. What is Sedna's equilibrium temperature at aphelion?
    d. Could Sedna, like Pluto, retain an $N_2$ atmosphere? Explain your reasoning.

e. The sublimation temperature of $N_2$ is 58K. If Sedna had significant surface $N_2$, would it be expected to be gaseous or frozen out on the surface at perihelion?

5. The Earth's atmosphere has a pressure scale height of $H \approx 8$ km.
   a. Compute the elevation at which the air pressure is 50% of the pressure at sea level.
   b. Look up the elevation of Mt. Everest, and provide your reference.
   c. Compute the ratio of the air pressure at the summit of Mt. Everest to that at sea level.
6. The estimated heat flow from the interior of Io due to tidal heating by Jupiter is about $10^{14}$ Watts. Io has a mean radius of 1821 km, an albedo of 0.63, and is at a distance of 5.2 AU from the Sun.
   a. What is the flux of sunlight absorbed by Io in W $m^{-2}$?
   b. What is the flux of internal heat escaping from Io in W $m^{-2}$?
   c. Does Io radiate more or less energy than it receives from the Sun? By how much?
7. Cometary nuclei are very dark, with an average albedo of A=0.05. Comets are also rapidly rotating. Once the comet nucleus gets close enough to the Sun that its surface temperature reaches T=150 K, its ices begin to sublime and form a gaseous coma. At what distance in AU from the Sun does this occur?
8. The gravitational binding energy, *U*, of a spherical planet with uniform density is given by

$$U = \frac{3}{5}\frac{GM^2}{R}$$

   where *M* and *R* are the planet's mass and radius, respectively. This is only a small (~10%) *underestimate* of the binding energy computed using a realistic density profile for a terrestrial planet, so it suffices for our purposes.
   a. What is the gravitational binding energy of the moon in Joules?
   b. Compare your estimate in part (a) with the total energy output of the Sun. How many days of the Sun's full energy output is equivalent to the binding energy of moon?
   c. Repeat parts (a) and (b) for Mars, Mercury, and the Earth.