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ANGULAR RADII OF STARS VIA MICROLENSING

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ABSTRACT

We outline a method by which the angular radii of giant and main-sequence stars located in the Galactic bulge can be measured to a few percent accuracy. The method combines comprehensive ground-based photometry of caustic-crossing bulge microlensing events, with a handful of precise ($\sim$10 μas) astrometric measurements of the lensed star during the event, to measure the angular radius of the source, θ_*. Dense photometric coverage of one caustic crossing yields the crossing timescale Δt. Less frequent coverage of the entire event yields the Einstein timescale $t_{\rm E}$ and the angle ϕ of source trajectory with respect to the caustic. The photometric light-curve solution predicts the motion of the source centroid up to an orientation on the sky and overall scale. A few precise astrometric measurements therefore yield $\theta_{\rm E}$, the angular Einstein ring radius. Then the angular radius of the source is obtained by $\theta_* = \theta_{\rm E}(\Delta t/t_{\rm E})\sin\phi$. We argue that the parameters $t_{\rm E}$, Δt, ϕ, and $\theta_{\rm E}$, and therefore θ_*, should all be measurable to a few percent accuracy for Galactic bulge giant stars using ground-based photometry from a network of small (1 m class) telescopes, combined with astrometric observations with a precision of $\sim$10 μas to measure $\theta_{\rm E}$. We find that a factor of $\sim$50 times fewer photons are required to measure $\theta_{\rm E}$ to a given precision for binary lens events than for single-lens events. Adopting parameters appropriate to the *Space Interferometry Mission* (*SIM*), we find that $\sim$7 minutes of *SIM* time is required to measure $\theta_{\rm E}$ to $\sim$5% accuracy for giant sources in the bulge. For main-sequence sources, $\theta_{\rm E}$ can be measured to $\sim$15% accuracy in $\sim$1.4 hr. Thus, with access to a network of 1 m class telescopes, combined with 10 hr of *SIM* time, it should be possible to measure θ_* to 5% for $\sim$80 giant stars, or to 15% for roughly seven main-sequence stars. We also discuss methods by which the distances and spectral types of the source stars can be measured. A by-product of such a campaign is a significant sample of precise binary lens mass measurements.

Subject headings: astrometry — binaries: general — gravitational lensing — stars: fundamental parameters

1. INTRODUCTION

Although of fundamental importance to stellar astrophysics, precise measurements of angular radii are generically difficult to acquire routinely and in a model-independent way. Classical direct methods of measuring stellar radii include lunar occultations, interferometry, and eclipsing binaries. Lunar occultation measurements yield precise angular radii (see Richichi et al. 1999 and references therein), but the number of stars to which this technique can be applied is limited. The number of direct measurements using interferometers has recently increased dramatically with advent of, e.g., the Palomar Testbed Interferometer (van Belle et al. 1999; Colavita et al. 1999) and the Navy Prototype Optical Interferometer (Armstrong et al. 1998; Nordgren et al. 1999) and is likely to continue to increase as more technologically advanced interferometers come on line. Unfortunately, both lunar occultation and interferometric angular diameter measurements have traditionally been primarily limited to nearby, evolved stars. Angular radii of main-sequence stars can be determined using detached eclipsing binaries (see e.g., Popper 1998); however, the large amount of data (both photometric and spectroscopic) required to yield accurate radii determinations makes this method prohibitive. Thus, of the $\sim$300 direct, precise angular diameter measurements compiled by van Belle (1999), the overwhelming majority, $\sim$85%, are of evolved stars. Finally, it will be difficult to acquire a large sample of angular radii determinations of stars with metallicity considerably smaller than solar using these methods, because of the paucity of metal-poor stars in the local neighborhood.

Here we present a method, based on a suggestion by Paczyński (1998), of measuring angular radii of stars that overcomes some of the difficulties inherent in the classical methods. This method employs the extraordinary angular resolution provided by caustics in gravitational microlensing events, and as such it is yet another in the growing list of applications of microlensing to the study of stellar astrophysics (see Gould 2001 for a review). The original suggestion of Paczyński (1998) was to invert the method of Gould

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(1994) for measuring the relative source lens proper motion $\mu_{\rm rel}$ in microlensing events. If the lens transits the source in a microlensing event, precise photometry can be used to determine the time it takes for the lens to transit one source radius, $t_* = \theta_*/\mu_{\rm rel}$, where θ_* is angular radius of the source. An estimate of θ_*, using an empirical color–surface brightness relation, together with a measurement of the flux of the source, can then be used to estimate $\mu_{\rm rel}$, which Gould (1994) argued could be used to constrain the location of the lens. However, as Paczyński (1998) pointed out, it is possible to independently measure the angular Einstein ring radius of the lens,

$$\theta_{\rm E} = \sqrt{\frac{4GM}{Dc^2}}\,, \tag{1}$$

by making precise astrometric measurements of the centroid shift of the source during the microlensing event using, e.g., the *Space Interferometry Mission* (*SIM*).[3] Here M is the mass of the lens, D is defined by $D \equiv D_{\rm os}D_{\rm ol}/D_{\rm ls}$, and $D_{\rm os}$, $D_{\rm ol}$, and $D_{\rm ls}$ are the distances between the observer and source, observer and lens, and lens and source, respectively. Since $\mu_{\rm rel} = \theta_{\rm E}/t_{\rm E}$, by combining the measurement of $\theta_{\rm E}$ with the Einstein timescale $t_{\rm E}$ of the event determined from the light curve, it is possible to measure θ_* for the source stars of microlensing events. We show that, with reasonable expenditure of resources, it should be possible to measure angular radii of a significant sample ($\sim$80) of giant stars in the bulge to an accuracy of $\lesssim$5%, or roughly seven main-sequence stars to an accuracy of $\lesssim$15%. Limb-darkening determinations should also be possible for the majority of the sources, and most will be relatively metal-poor as compared to those for which angular radii determinations are currently available.

Although measurements of θ_* can be made using single-lens events, in § 2 we argue that this method is better suited to caustic-crossing binary lens events, which are more common, easier to plan for, and considerably less resource intensive than source-crossing single-lens events. We describe in some detail the basic method of measuring θ_* for the source stars of caustic-crossing binary lens events in § 3, including a discussion of the expected errors on the individual parameters that enter into the measurement. We discuss various subtleties, complications, and extensions to the method in § 4 and also present an estimate of the number of θ_* measurements that might be made in this way. Finally, we summarize and conclude in § 5.

2. BINARY VERSUS SINGLE-LENS EVENTS

The primary requirement to be able to measure $\theta_{\rm E}$ in a microlensing event is that the source should be resolved by the gravitational lens. This effectively means that the source must cross a caustic in the source plane. Caustics are the set of positions in the source plane where the determinant of the Jacobian of the lens mapping from source to lens plane vanishes and where the magnification is therefore formally infinite. Large gradients (with respect to source position) in the magnification exist near caustics, enabling the resolution of the source. Generically, microlenses come in two classes: single and binary lenses. Here "binary lens" means a lens system composed of two masses with angular separation of the order of the angular Einstein ring radius of the system. Very close and very wide binaries act essentially as single lenses. Single lenses have a caustic that consists of a single point at the position of the lens. In these cases, the magnification close to the caustic diverges as the inverse of the distance to the caustic. In contrast, the caustics of binary lenses are extended and can cover a significant fraction of the Einstein ring. The caustics of binary lenses generically consist of two types of singularities, folds and cusps.[4] Near a fold, the caustic is well described by generic linear fold singularities, for which the magnification locally diverges inversely as the square root of the distance to the caustic (Schneider & Weiss 1986; Schneider, Ehlers, & Falco 1992; Gaudi & Petters 2002a). Cusps are points where two fold caustics meet, and the magnification for cusps locally diverges roughly inversely as the distance to the cusp point, similar to the magnification pattern near the point caustic of a single lens (Schneider & Weiss 1992; Schneider et al. 1992; Gaudi & Petters 2002b). The fact that the magnification near the point caustic of a single lens or near a cusp diverges inversely as the distance to the singularity, rather than as the square root of the distance to the singularity as with folds, means that for a given source size, the "resolving power" of fold caustic crossings is less than that of single-lens or cusp crossings.

Although single-lens events are better suited to studies of stellar atmospheres, they are much less useful for measuring the sizes of stars for three main reasons. They are generally rarer than fold caustic crossings, their crossings cannot be predicted in advance, and their centroid motion is more difficult to measure.

Caustic-crossing binaries comprise roughly $f_{\rm cc} = 7\%$ of all events toward the Galactic bulge (Alcock et al. 2000a; Udalski et al. 2000). Of the remaining $1 - f_{\rm cc}$ events, which we conservatively assume to be due to single lenses, only a fraction $\theta_*/\theta_{\rm E}$ will exhibit caustic crossings. Therefore, the expected ratio of binary to single events for which the source is resolved (and thus measurement of θ_* is possible) is

$$\Gamma_{b/s} \simeq \frac{\theta_{\rm E}}{\theta_*}\frac{f_{\rm cc}}{1-f_{\rm cc}} \sim 4\left(\frac{R_*}{10\ R_\odot}\right)^{-1}, \tag{2}$$

where R_* is the physical radius of the source, and we adopt $D_{\rm ol} = 6$ kpc, $D_{\rm os} = 8$ kpc, and $M = 0.3\ M_\odot$ for the scaling relation on the extreme right-hand side. Although not overwhelming for giant sources, for main-sequence sources we expect at least an order of magnitude more binary lensing events for which the source is resolved. The ratio of fold to

[3] See http://sim.jpl.nasa.gov.

[4] For binary lenses, higher order, beak-to-beak singularities can exist for specific combinations of the binary lens mass ratio and angular separation in units of $\theta_{\rm E}$. However, folds and cusps are the only stable singularities of any lens system (Petters, Levine, & Wambsganss 2000). Beak-to-beak singularities are unstable in the sense that, for infinitesimally small variations in the lens parameters, a beak-to-beak singularity disintegrates into two cusp-type singularities. Thus, the set of parameters for which beak-to-beak singularities are expected is formally sparse and practically small. Interestingly, Alcock et al. (2000a) suggested that an observed event may have been due to a source crossing a beak-to-beak singularity in a binary lens, although Alcock et al. (2000b) seem to favor the interpretation that this event is due to a background supernova.

cusp crossings is roughly

$$\Gamma_{f/c} \simeq \frac{\theta_{\rm E}}{\theta_*} N_{\rm cusp}^{-1} \sim 55 N_{\rm cusp}^{-1} \left(\frac{R_*}{10 R_\odot}\right)^{-1}, \qquad (3)$$

where $N_{\rm cusp}$ is the number of cusps, which is either 6, 8, or 10 for a binary lens, depending primarily on d. Thus, the overwhelming majority of events for which it will be possible to measure θ_* will be fold caustic–crossing binary lens events.

Binary lens fold caustic–crossing events also have the advantage that the second caustic crossing can be predicted in advance. This is because fold caustic crossings always come in pairs, and it is typically easy to tell, even with sparse sampling, that the first caustic crossing has occurred. Then more frequent sampling can be used to monitor the rise to the second caustic, in principle enabling the prediction of the time of the second crossing a day or more in advance (Jaroszyński & Mao 2001) and allowing the marshaling of the resources necessary to obtain the dense coverage of the second crossing needed to measure the crossing time Δt of the source (see § 3.1). In contrast, a single-lens caustic crossing can only reliably be "predicted" at about the time it begins.

Finally, it is considerably harder to measure $\theta_{\rm E}$ for caustic-crossing single-lens events than for binary lensing events. Single-lens events have a maximum absolute centroid shift relative to the unlensed source position of $\theta_{\rm E}/8^{1/2}$, whereas binary lensing events can exhibit large variations of size $\sim\theta_{\rm E}$ or more when the source crosses the caustic (Han, Chun, & Chang 1999). Therefore, considerably more time will generally be required to determine $\theta_{\rm E}$ to given accuracy for single-lens events than for binary lens events. Since the astrometric measurements essentially require the capabilities of *SIM* (or some similarly precious instrument), it is highly desirable to minimize the amount of time spent on this step.

Given the above arguments, we conclude that fold caustic–crossing binary lensing events are the most suitable for use in routinely measuring θ_*. We therefore focus on this case for the remainder of the discussion; however, we briefly revisit single-lens and cusp-crossing events in § 4.1.

3. THE METHOD

Consider a binary lens event in which the source crosses a simple linear caustic.[5] Defining Δt as (one-half) the time it takes for the source to completely traverse the caustic and ϕ as the angle between the source trajectory and the tangent to the caustic at the crossing point, then the time for the lens to cross the angular radius of the source is $t_* = \Delta t \sin\phi$. However, we also have $t_* = \theta_*/\mu_{\rm rel}$, where, again, $\mu_{\rm rel}$ is the relative source lens proper motion. Combining these expressions, we have $\theta_* = \mu_{\rm rel}\Delta t \sin\phi$. Using the definition of $\mu_{\rm rel}$, we can write the angular source radius θ_* as the following function of observables,

$$\theta_* = \theta_{\rm E} \frac{\Delta t}{t_{\rm E}} \sin\phi\,. \qquad (4)$$

The process of θ_* measurement can therefore be subdivided into three basic steps:

1. measurement of the caustic-crossing timescale Δt from a single photometrically well-resolved caustic crossing;
2. measurement of the angle ϕ and timescale $t_{\rm E}$ from the global fit to the binary lens light curve;
3. measurement of the angular Einstein ring radius $\theta_{\rm E}$ using precise astrometric measurements of the source centroid.

Each of these steps is illustrated schematically in Figure 1.

In the following subsections, we consider each of these steps in more detail. We outline the basic requirements for the measurement of each of the four parameters (Δt, ϕ, $t_{\rm E}$, and $\theta_{\rm E}$) and the expected accuracy with which each can be determined, assuming reasonable expenditure of observing resources.

3.1. *Measuring* Δt

When the source is interior to a binary lens caustic, five images are created. As the source approaches a fold caustic, two of these images brighten and merge in a characteristic way and eventually disappear when the source completely exits the caustic. In contrast to the significant brightening of the two images associated with the fold, the remaining three images generally vary only slowly over the timescale of the crossing. All fold caustics locally have this generic behavior, and the magnification $A(t)$ of the source near a caustic crossing as a function of time is typically well fitted by the functional form

$$A(t) = \frac{t_r}{\Delta t} G_0 \frac{t - t_{\rm cc}}{\Delta t} + A_{\rm cc} + \omega(t - t_{\rm cc}) \qquad (5)$$

(Albrow et al. 1999b), where t_r is the effective rise time of the caustic, which is related to the local derivatives of the lens mapping (Petters et al. 2001; Gaudi & Petters 2002a), $t_{\rm cc}$ is the time when the center of the source crosses the caustic, $A_{\rm cc}$ is the magnification of all the images unrelated to the fold caustic at $t = t_{\rm cc}$, ω is the slope of the magnification of these images as a function of time, and $G_0(x)$ can be expressed in terms of complete elliptic integrals of the first and second kind (Schneider & Weiss 1987). Note that equation (5) is only formally appropriate for a simple linear fold caustic. Thus, for a well-sampled fold caustic crossing, Δt can be determined essentially independently of the global geometry of the event, and indeed without reference to the photometric data away from the crossing itself. In practice, the magnification is not directly observable, but rather the flux $F(t)$ as a function of time. $F(t)$ takes on a form similar to equation (5), but with a slightly different parameterization (see Albrow et al. 1999b).

Equation (5) assumes a uniform source. This will likely be a poor approximation in optical bands, and assuming uniform source in the presence of limb darkening, it may result in a systematic underestimate of Δt and therefore θ_*, since the effect of limb darkening can partially compensated for by a smaller (dimensionless) source size, at least for poorly sampled light curves. However, for well-sampled caustic

[5] A caustic can generally be approximated as a simple linear fold when the curvature of the caustic is everywhere small on angular scales of $\mathcal{O}(\theta_*)$ and when the angle of incidence of the source trajectory to the caustic is not small. This approximation will break down when the source is large compared to the overall size of the caustic, when the source crosses near a cusp, or when the source "straddles" the caustic for a long time because of a small incidence angle. Although it will still be possible to measure θ_* for such events, the relation between the observables and θ_* is less straightforward. We discuss such cases in § 4.1.

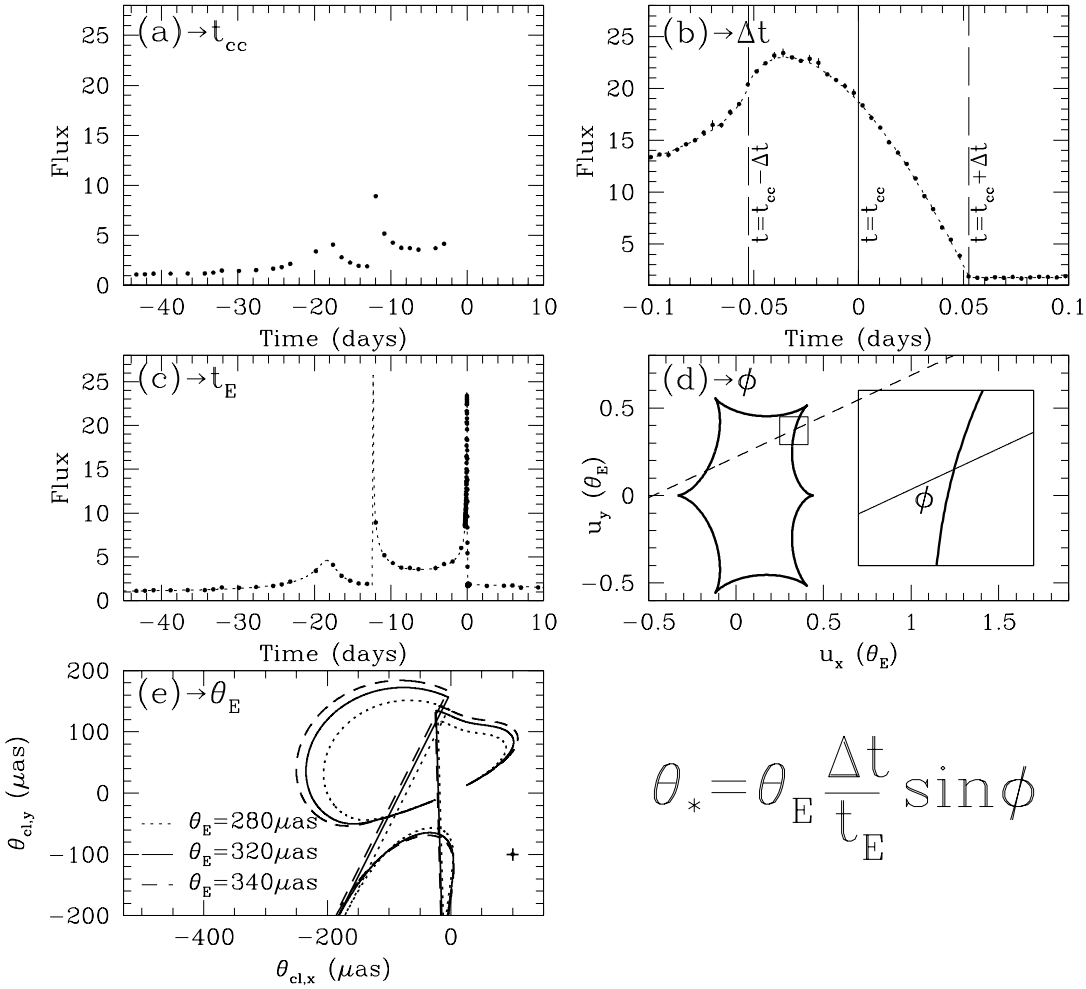

FIG. 1.—Schematic illustration of the various steps involved in measuring the angular radius of the source star of a caustic-crossing binary microlensing event. (*a*) Infrequent ($\sim$1 day^{-1}) observations of bulge microlensing events from a single site reveal a caustic-crossing event in real time, enabling a prediction for, and intense photometric monitoring of, the second caustic crossing. (*b*) Fitting the photometry near the second crossing to a generic fold caustic model yields the caustic-crossing timescale Δt. (*c*, *d*) Fitting the entire photometric data set to a binary lens model yields the Einstein timescale $t_{\rm E}$ of the event, as well as the angle ϕ of the trajectory with respect to the caustic. (*e*) The photometric solution predicts the motion of the centroid of the source up to an unknown scale and orientation on the sky. A few precise astrometric measurements can then be used to determine $\theta_{\rm E}$, the angular Einstein ring radius. The cross shows error bars of 10 μas. The angular source size is then given by $\theta_* = \theta_{\rm E}(\Delta t/t_{\rm E}) \sin\phi$.

crossings and $\sim$1% photometric accuracy, it should generally be possible to accurately measure both the source size and limb-darkening coefficient(s) (Rhie & Bennett 1999). Such data quality is readily achievable; indeed, independent limb-darkening and (dimensionless) source size measurements have been made for the source stars of at least five microlensing events (Albrow et al. 1999a; Afonso et al. 2000; Albrow et al. 2000, 2001a; An et al. 2002). A generalized form of equation (5) that includes simple limb darkening can be found in Albrow et al. (1999b) and Afonso et al. (2001).

By fitting equation (5) (or a generalized form of it) to the light curve near a well-sampled fold caustic crossing, one can derive the parameters t_r, $t_{\rm cc}$, $A_{\rm cc}$, ω, and Δt. The parameters $t_{\rm cc}$, $A_{\rm cc}$, and t_r can subsequently be used to constrain the global solution to the entire light curve (see Albrow et al. 1999b). However, of primary interest here is the parameter Δt, whose value is essentially independent of the global solution; see Figure 1*b*. This means that Δt and the parameters determined from the global solution, $t_{\rm E}$ and ϕ, will be essentially uncorrelated.

In order to be able to determine Δt from the caustic-crossing data alone, the caustic crossing must be well sampled, so that the parameters in equation (5) can be well constrained. Practically, this requires forewarning of the caustic crossing. Fortunately, this is generally possible with only photometry available from the collaborations that survey the Galactic bulge and find and "alert" (i.e., notify other investigators of) the microlensing events in real time (EROS, Afonso et al. 2001; MOA, Bond et al. 2001; OGLE, Udalski et al. 2000, Woźniak et al. 2001), although improved predictions would be possible with continuous photometry (Jaroszyński & Mao 2001). Thus, no additional resources need to be invested to predict caustic crossings. However, as we discuss in § 3.2, some additional sampling of the overall light curve may be needed to constrain $t_{\rm E}$ and ϕ and determine a unique global solution.

The accuracy with which Δt can be determined for a given caustic crossing will depend not only on the intrinsic parameters of the caustic crossing, but also on the sampling rate and photometric accuracy near the caustic crossing, which in turn will depend on factors that tend to vary in a

stochastic manner, such as weather and blending. Therefore, it is not very useful to attempt to quantify the expected errors on Δt for an idealized observing setup. However, we can obtain an order-of-magnitude estimate for $\sigma_{\Delta t}$ by examining measurements of Δt from published analyses of observed caustic-crossing events. We discuss these determinations more thoroughly in § 3.4. Typically, well-covered caustic crossings yield fractional errors of $\sigma_{\Delta t}/\Delta t \sim 1\%$.

3.2. *Measuring t_E and ϕ*

The parameters t_E and ϕ are determined from the global solution to the overall geometry of the binary lens light curve. The relationship between the salient features of an observed light curve and the canonical parameters of a binary lensing event are generally not obvious or straightforward. This fact generally makes fitting an observed light curve, and thus inferring the parameters t_E and ϕ, quite difficult (see Albrow et al. 1999b for a thorough discussion). Although many methods have been proposed to overcome these difficulties, the lack of obvious correspondence between these parameters of interest and the light-curve morphology generally implies that it is difficult to make general statements about the kinds of observations that are needed to reliably measure t_E and ϕ. However, we can make some generic comments. For caustic-crossing binary lenses, one potential observable is the time δt_{cc} between caustic crossings. Between caustic crossings, the magnification is typically considerably larger than outside the caustic, and therefore even with sparse ($\sim$1 day) sampling, it should be possible to determine to reasonable precision the time of the first caustic crossing, provided that data exist before the first crossing. The requisite photometry will generally be acquired by the survey collaboration(s). Once the binary lens event is detected, more frequent photometry can be acquired by follow-up collaborations (PLANET, Albrow et al. 1998; MPS, Rhie et al. 2000), thus mapping the U-shaped curve between the caustic crossings. This shape, combined with information from cusp approaches (or lack thereof) just outside the caustic, provides information about the shape of the caustic and the trajectory of the source through it. From this, the angle ϕ of the trajectory with respect to the caustic can be derived and also the distance d_{cc} between the caustic crossings in units of θ_E. Then the timescale is given, by $t_E = t_{cc}/d_{cc}$.

Although the above analysis is highly trivialized, it does suggest that the following steps should be taken to ensure an accurate measurement of t_E and ϕ. First, it is important to constrain the time of both caustic crossings to reasonable precision. This means that the survey collaborations should sample on timescales no less than a few days. Second, follow-up photometry should be initiated relatively soon after the first crossing, to measure the shape of the intracaustic light curve reasonably well. This is also necessary in order to predict the second caustic crossing (Jaroszyński & Mao 2001). Furthermore, the follow-up photometry should continue past the second caustic crossing, to detect cusp approaches or cusp crossings (or the lack thereof).

Because of the difficulties inherent in fitting binary lens light curves, it would be extremely difficult to attempt to predict the expected errors on t_E and ϕ for a hypothetical observing scenario. Furthermore, the complicated relation between observables and parameters implies that these errors are likely to depend strongly on the geometry of the event and light-curve coverage, and therefore such "predictions" would not be very useful. However, we would like to have an order-of-magnitude estimate for the expected errors, given reasonable light-curve coverage. From determinations of these parameters in published caustic-crossing events, we can expect fractional accuracies of a few percent, provided that the light curve is well covered in the sense outlined above. However, if the light-curve coverage is incomplete, then errors of $\gtrsim 20\%$ are expected. See § 3.4.

3.3. *Measuring θ_E*

A global solution to the entire light curve effectively requires the specification of the vector position of the source $\boldsymbol{u}(t)$ as a function of time in units of θ_E and the topology of the lens, i.e., the mass ratio q and projected separation d in units of θ_E. These parameters yield not only the total magnification A of all the images as a function of time, but also the individual image positions $\boldsymbol{\varphi}_i$ and magnifications A_i as a function of time, Therefore, it is also possible to predict the centroid $\boldsymbol{\varphi}_{cl}$ of all the individual microimages,

$$\boldsymbol{\varphi}_{cl} = \frac{\sum_i A_i \boldsymbol{\varphi}_i}{A} \ . \tag{6}$$

Note that $\boldsymbol{\varphi}_{cl}$ is the centroid shift with respect to the lens position. It is customary to consider the centroid shift with respect to the unlensed source position, $\delta\boldsymbol{\varphi}_{cl} \equiv \boldsymbol{\varphi}_{cl} - \boldsymbol{u}$. The unlensed source position $\boldsymbol{u}$, which is comprised of the parallax and proper motion of the source in some astrometric frame, can be determined via measurements of the unlensed motion of the source. Since the astrometric effects fall off very slowly (as u^{-1}), these must be obtained many t_E after the event is over. In fact, such measurements are not strictly needed in order to measure θ_E (but may be desirable for other reasons; see § 4.3). Rather, one can simultaneously fit astrometric measurements during the course of the event for the relative position and proper motion of the source (in order to establish a local astrometric reference frame) and the offset induced by microlensing.

The centroid shift $\boldsymbol{\varphi}_{cl}$ is in units of θ_E, and its components are oriented with respect to the projected binary lens axis, whose orientation α on the sky is unknown. The observable centroid is

$$\boldsymbol{\theta}_{cl}(t) \equiv \theta_E \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \boldsymbol{\varphi}_{cl}(t) \ . \tag{7}$$

Alternatively, one can combine α and θ_E and simply consider the vector $\boldsymbol{\theta}_E$. Thus, the global solution to the photometric light curve yields not only t_E and ϕ, but also a prediction for the astrometric curve, up to an unknown orientation and scale θ_E.[6] Therefore, by making a series of astrometric measurements $\boldsymbol{\theta}_{cl}(t)$ at several different times during the course of the event, one can determine θ_E and α via equation (7) and using the predictions for $\boldsymbol{\varphi}_{cl}(t)$ from the photometric solution.

The accuracy with which θ_E can be determined will depend on the geometry of the event, the time of the astrometric measurements, and the time span between the measurements. Furthermore, as we discuss, there are, in reality,

[6] Technically, this is only true for an astrometric observer that is spatially coincident with the photometric observer. This will not be true for astrometric observations with *SIM*, which will be in an Earth-trailing orbit. We discuss this complication in § 4.2.2.

additional parameters that must be determined from the astrometric data. We therefore perform a Monte Carlo simulation to estimate the accuracy with which θ_E can be recovered. We explore whether the various parameters can be measured independently or are degenerate with the measurement of θ_E, and we study the range of fractional uncertainties in measuring θ_E for a full ensemble of binary lenses.

Our simulation consists of the following elements: a Galactic model and a model of the ensemble of lenses to generate an ensemble of microlensing events, an observational strategy, and a fit of these observations to a set of parameters including θ_E.

Our ensemble of lenses is identical to that of Graff & Gould (2002). Briefly, we draw sources and lenses from a self-lensing isothermal sphere 8 kpc from the observer and with two-dimensional velocity dispersion of 220 km s^{-1}. Both masses in the binary lens are chosen from the remnant mass function of Gould (2000). We pick a flat distribution in $\log d$, the logarithm of the dimensionless binary separation. The path of the source through the lens geometry is chosen randomly, with a uniform distribution of angular impact parameter $b\theta_E$, and we only consider paths that cross caustics.[7]

As discussed in Graff & Gould (2002), in this ensemble of events there are many more events with a short time between caustic crossings, the *caustic interior time* t_{int}, than is observationally detected by the MACHO and OGLE observing groups. This led these authors to suggest that most events with a short caustic interior time are not detected as caustic-crossing binaries and to define a caustic-crossing detection efficiency $\mathscr{E}_b(t_{int})$ akin to the standard single-lens detection efficiency $\mathscr{E}_s(t_E)$.

The observational strategy is relatively unimportant, as long as there are astrometric measurements on either side of a caustic. Although it is conceivable that observations might be scheduled at particularly favorable times, such as times of maximum magnification or maximum displacement of the image centroid, it is likely that the telescope measuring the astrometric displacement will be oversubscribed. A simpler strategy would be to schedule periodic observations in advance. We have assumed that observations will be made every 4 days, with a 24 hr delay after the event is recognized as a caustic-crossing binary, i.e., after the first caustic crossing. That is, the first observation comes 1–5 days after the first caustic crossing. We assume that observations are made for a total of 36 days, which corresponds to approximately $2t_E$ for the median event timescale. Practically, observations should continue until after the second caustic crossing, and the total number of observations should be at least as large as the number of parameters to be constrained.

As we discuss in § 3.4 (see also Table 1), well-covered binary lenses can be fitted photometrically with small errors on the parameters. Thus, we have assumed that all the parameters that can be fitted from a single photometric telescope are determined.

Given a binary microlensing event from our ensemble, we use our observational strategy to create a series of photometric and astrometric measurements, $A(t)$ and $\boldsymbol{\theta}_{cl}(t)$, respectively, which we can combine into a single list of measurements M_k, each with uncertainty σ_k. Using the Fisher matrix technique (see, e.g., Gould & Welch 1996), we determine the covariance matrix c_{ij} of the errors

$$c \equiv b^{-1}, \quad b_{ij} = \sum_k \sigma_k^{-2} \frac{\partial M_k}{\partial a_i} \frac{\partial M_k}{\partial a_j} \ . \tag{8}$$

Here the a_i are the various parameters being fitted. The error in parameter a_i is simply $\sigma_{a_i} = c_{ii}^{1/2}$.

We assumed that the photometric uncertainty σ_k of the interferometric telescope is photon noise–dominated and that the total telescope time, aperture, efficiency, filter width, and source brightness are such that a total of $N = 60,000$ photons would be detected from an unmagnified source for a total photometric signal-to-noise ratio of 250.[8] We assume that the fractional photometric accuracy is simply $N^{-1/2}$ and that the astrometric uncertainty is $\sigma_\theta = N^{-1/2}\theta_f$, where θ_f is the width of the point-spread function, or in the case of an interferometer, the fringe separation. Here we have assumed $\theta_f = 2.5$ mas. It is trivial to scale our results to brighter sources or larger telescopes: the fractional uncertainty in θ_E is simply proportional to $N^{-1/2}$.

We always fitted for the four parameters required to establish a local astrometric frame and for θ_E and α, the size scale and orientation, respectively, of the microlensing excursion. We also assumed that the satellite that measures the astrometric motion is 0.2 AU from the (ground-based) photometric measurements that fix the lens parameters. Thus, we can simultaneously fit for $\tilde{r}_E$, the projected Einstein ring radius, in the manner of Graff & Gould (2002). In

[7] This is in contrast to the usual technique for single lenses, in which events are chosen from a uniform distribution in b, but are weighted toward large-θ_E events by multiplying the mass function by $M^{1/2}$.

[8] Note that a telescope with an overall efficiency of ~30% and diameter A_T collects $\sim 100(A_T\,\mathrm{m}^{-1})^2$ photons s^{-1} at $I = 18$.

TABLE 1

Fractional Errors on Parameters from Observed Events

Event Name	$\sigma_{\Delta t}/\Delta t$ (%)	σ_ϕ/ϕ (ϕ)	σ_{ρ_*}/ρ_* (%)	σ_{t_E}/t_E (%)	I (mag)	References	Comments
MACHO 98-SMC-1	0.9	32 (36°.9)	...	23	22.1	1, 2	Fold crossing
OGLE-1999-BUL-23	0.5	0.2 (56°.1)	...	1	18.1	3	Fold crossing
MACHO 95-BLG-30	...	...	0.1	...	13.4	4	Single lens
MACHO 97-BLG-28	...	...	0.3	...	15.6	5	Cusp crossing
MACHO 97-BLG-41	...	...	5	...	16.8	6	Rotating binary
EROS 2000-BLG-5	...	...	0.8	...	16.6	7	Parallax effects

References.—(1) Albrow et al. 1999b; (2) Afonso et al. 2000; (3) Albrow et al. 2001a; (4) Alcock et al. 1997; (5) Albrow et al. 1999a; (6) Albrow et al. 2000; (7) An et al. 2002.

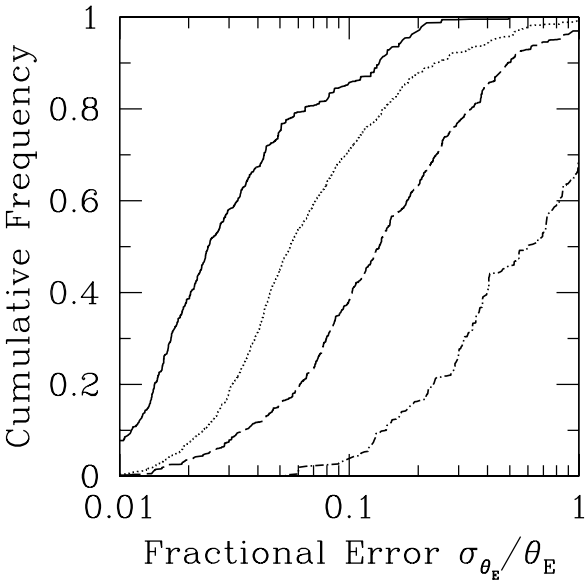

FIG. 2.—Cumulative distribution of expected errors in θ_E in a Monte Carlo simulation, assuming a contribution of blended light of 0% (no blending; *solid line*), 1% (*dotted line*), 10% (*dashed line*), and 90% (*dot-dashed line*). See text.

addition to this basic fit, we also considered blending from luminous lenses and binary sources, which requires several additional parameters. We discuss parallax and blending in §§ 4.2.2 and 4.2.3, respectively.

Our basic results are summarized in the leftmost curve in Figure 2. We see that in the absence of blending, θ_E can be determined with less than 10% uncertainty in 85% of events. The median error is $\sigma_{\theta_E}/\theta_E \simeq 2.4\%$. This is comparable to the uncertainty found by Gould & Salim (1999) for single-star events, but with 50 times as many photons as we have assumed here. Thus, it is much easier to measure θ_E for caustic-crossing binary lenses than for single lenses.

3.4. *Expected Fractional Errors on θ_**

From the expression for θ_* (eq. [4]), and assuming the errors in θ_E, t_E, Δt, and ϕ are small and uncorrelated, the fractional error in θ_* is given by

$$\frac{\sigma_{\theta_*}}{\theta_*} = \left[\left(\frac{\sigma_{\theta_E}}{\theta_E}\right)^2 + \left(\frac{\sigma_{t_E}}{t_E}\right)^2 + \left(\frac{\sigma_{\Delta t}}{\Delta t}\right)^2 + \sigma_\phi^2 \cot^2\phi\right]^{1/2}, \quad (9)$$

where σ_{θ_E}, σ_{t_E}, $\sigma_{\Delta t}$, and σ_ϕ are the uncertainties in θ_E, t_E, Δt, and ϕ, respectively, and σ_ϕ is in radians.

To date, there exist in the published literature 10 microlensing events for which the source star was well resolved. Unfortunately, for four of the events, those presented in Alcock et al. (2000a), no estimate of the errors of the derived parameters is given. We therefore cannot use these events to explore the expected magnitudes of σ_{t_E}, $\sigma_{\Delta t}$, and σ_ϕ.

For the six events for which errors on the relevant fit parameters were given or derivable, only two of them are generic binary lens fold caustic–crossing events. The errors σ_{t_E}, $\sigma_{\Delta t}$, and σ_ϕ for these two events, MACHO 98-SMC-1 and OGLE-1999-BUL-23, are presented in Table 1. For MACHO 98-SMC-1, these errors have been determined from the ensemble of solutions presented in Albrow et al. (1999b), which were fits to the PLANET collaboration photometry, which only covered the last half of the event, whereas the analysis of the combined photometry of the EROS, MACHO, MPS, OGLE, and PLANET collaborations (Afonso et al. 2000) yields considerably smaller errors. We concentrate on the results of Albrow et al. (1999b) here in order to demonstrate the kinds of errors that result from incomplete light-curve coverage. Note that $\sigma_{\Delta t} \sim 1\%$, which is not surprising, since the accuracy with which Δt can be determined depends almost exclusively on the photometric coverage near the caustic crossing. However, the errors on the global parameters t_E and ϕ are quite large, $\sigma_{t_E} \sim 20\%$ and $\sigma_\phi \sim 30\%$. This is due to the fact that the PLANET photometry only covered the latter half of the event and contained no data prior to or during the first caustic crossing. Therefore, the global geometry was quite poorly constrained with their data alone. Such incomplete coverage would clearly jeopardize a precise measurement of θ_*. This is in contrast to OGLE-1999-BUL-23, for which both PLANET and OGLE obtained data before the first caustic crossing. In this case, it was possible to measure Δt, t_E, and ϕ to better than 1% (Albrow et al. 2001a).

The remaining four include a single-lens event (MACHO 95-BLG-30; Alcock et al. 1997), a binary lens event in which the source crossed a cusp (MACHO 97-BLG-28; Albrow et al. 1999a), a binary lens event in which the rotation of the binary was detected (MACHO 97-BLG-41; Albrow et al. 2000), and a binary lens event for which both rotation and parallax effects were detected (EROS BLG-2000-5; An et al. 2002). In general, the previous discussion is not directly applicable to these events, as the information in the light curves is not easily decomposed into the parameters Δt, ϕ, and t_E. Nevertheless, in all four cases the dimensionless source size $\rho_* \equiv \theta_*/\theta_E$ was determined. In three of the cases, $\delta\rho_*/\rho_* \lesssim 1\%$. In one case, MACHO 97-BLG-41, $\delta\rho_*/\rho_* \sim 5\%$, primarily as a result of the fact that there exist only a handful of data points in which the source was resolved. Thus, although it is difficult to draw any general conclusions from these unique events, it is does seem likely that errors of $\lesssim 1\%$ are achievable for most types of events in which the source is resolved.

Also presented in Table 1 are the determined values of the I magnitude of the source, for all six events. At $I \sim 22$, the source star for MACHO 98-SMC-1 would be too faint to target with most upcoming interferometers, including *SIM*. The other events are primarily bulge clump giants ($I \sim 15$), for which accuracies of $\sigma_{\theta_E}/\theta_E \lesssim 5\%$ should be achievable with a reasonable amount of exposure time with upcoming interferometers and in particular with $\lesssim 8$ minutes of *SIM* time (see § 4.5 for estimates of the required exposure times for *SIM*). In these cases, the expected error on θ_* is typically dominated by σ_{θ_E}, and therefore we can expect $\sigma_{\theta_*}/\theta_* \sim \sigma_{\theta_E}/\theta_E \sim 5\%$. The source of OGLE-1999-BUL-23 is a G/K subgiant ($R_* \sim 3\ R_\odot$) and thus dimmer. The time required to achieve an accuracy of $\sigma_{\theta_E}/\theta_E \sim 5\%$ will be larger by a factor of ~16, or ~2 hr for *SIM*.

4. DISCUSSION

Our goal in § 3 was to capture the essence of the method of measuring θ_*, and the discussions were therefore somewhat oversimplified and glossed over several important points. In particular, we concentrated on fold caustic–crossing binary lens events toward the bulge, whereas measurements of θ_* should be possible in other, rarer, types of

events, such as cusp crossings, single-lens events, and possibly events toward the Magellanic Clouds. We also ignored various higher order effects that could, in principle, complicate the measurements. We therefore briefly discuss some of these complications and extensions. We also discuss the prospects for measuring the spectral type of the source and also its distance, in order to convert from angular radius θ_* to physical radius R_*. Finally, we present an example observing campaign aimed at measuring angular radii for a significant sample of sources, outlining the resources required and estimating the number of θ_* measurements that might be made per year for such a campaign.

4.1. *Single-Lens, Cusp-crossing, and Magellanic Cloud Events*

Although we have focused on fold caustic–crossing binary lens events toward the bulge, it is important to emphasize that θ_* can, in principle, be measured for other types of caustic-crossing events, such as cusp-crossing events, single-lens events, and all types of caustic-crossing events toward the Magellanic clouds. Indeed, in § 3.4 we discussed examples in the literature of a single-lens event and two cusp-crossing events for which a $\sim$5% measurement of θ_* would have been feasible.

In general, isolated cusp crossings, such as in MACHO 98-BLG-28, cannot be predicted in advance, and thus planning for such events is difficult, if not impossible. However, one will still have some advance warning of those cusp events that occur just after or in place of second fold caustic crossings. For such events, sufficient photometric coverage of the crossing should routinely be possible. In all cases, it is more difficult to disentangle the information arising from the cusp itself from the information from the global light curve. This generally implies that the analysis of these light curves will be more complicated; however, this does not necessarily preclude an accurate measurement of θ_*.

Single-lens events are less desirable simply because they require a factor of $\sim$50 times more astrometric observing time to achieve the same fractional accuracy in $\theta_{\rm E}$ as binary lens events. Since the astrometric observations are likely to be the most limited resource, this makes single-lens events considerably less attractive.

If it were possible to measure angular radii of stars in the Magellanic clouds (MCs), this would be quite interesting, because of the metal-poor nature of the stars. Unfortunately, there are several major hindrances to measuring θ_* for a substantial number of stars in the MCs. First, the event rates toward both the MCs are small, and a large number of stars must be monitored just to detect a few events per year. Therefore, the number of caustic-crossing events is quite low. To date, there have been only two caustic-crossing events toward the MCs: MACHO 98-SMC-1, which we discussed in § 3.4, and MACHO LMC-9. These events have source magnitudes of $V_S = 22.4$ (Afonso et al. 2000) and 21.4 (Alcock et al. 2000a), respectively, which brings up a second difficulty: *SIM* cannot follow source stars fainter than $V \sim 20$, so these two events could not have been used to measure the angular radii of their source stars. In fact, even if the entire LMC were monitored for microlensing, only $\sim$1 event yr^{-1} would have $V_S \lesssim 20$, and this event would be from an evolved star. The probability of a caustic-crossing event (either binary or single-lens) is smaller by at least an order of magnitude. The paucity of events and faintness of the source stars might be circumvented if sufficiently rapid target-of-opportunity times are available. In this case, it might be possible to use intrinsically fainter source stars, for which caustic-crossing events will be more common, and measure the astrometric displacement during the brief period of time when the source is highly magnified as it crosses the caustic. The maximum magnification of a source of dimensionless size ρ_* crossing a fold caustic is $A_{\rm max} \sim \rho_*^{-1/2}$. For main-sequence sources, $A_{\rm max} \gtrsim 30$, or more than 3 mag, and thus sources with $V_S \lesssim 23$ can briefly be brightened to *SIM* detectability. For example, the source star of MACHO 98-SMC-1 was brighter than $V \sim 18$ for about 7 hr during the second caustic crossing. Finally, even if the source does attain a sufficient brightness to be measurable by *SIM*, it remains to be seen whether the centroid varies sufficiently during this time to provide an accurate measurement of $\theta_{\rm E}$. This is especially difficult in light of the fact that typical value of $\theta_{\rm E}$ for self-lensing events toward the MCs are only an order of magnitude larger than *SIM*'s accuracy (Paczyński 1998; Gould & Salim 1999). In summary, it appears that it will be quite difficult to measure angular radii of stars in the MCs using this method, especially if the majority of the events seen toward these targets are due to self-lensing (Sahu 1994).

4.2. *Complications to $\theta_{\rm E}$ Measurement*

The method we have presented here is only interesting if it can feasibly be used to make precise θ_* measurements for a large number of sources with reasonable expenditure of resources. Since the requisite astrometric instruments are likely to be the most limited resource, it is crucial that accurate and unambiguous determinations of $\theta_{\rm E}$ be generically possible using a few astrometric measurements, when combined with the photometric light-curve solution. We have explained how a complete photometric solution *generally* leads to a prediction for the astrometric centroid shift up to an unknown scale $\theta_{\rm E}$ and orientation α on the sky. However, this is true only under a number of simplifying assumptions, including uniform motion of the observer, source, and lens, dark lenses, isolated sources, and unique global solutions. If one or more of these assumptions are violated, then the prediction for shape of the astrometric curve may not be unique, and thus the measurement of $\theta_{\rm E}$ may be compromised. We therefore discuss each of these complications and under what conditions they may be important.

4.2.1. *Binary Lens Degeneracies*

Binary lenses are characterized by two quantities: q, the mass ratio, and d, the instantaneous projected separation in units of $\theta_{\rm E}$. It has been demonstrated both theoretically (Dominik 1999a) and observationally (Afonso et al. 2000; Albrow et al. 2002) that certain limiting cases of binary lenses can exhibit extremely similar observable properties. In particular, Dominik (1999b) showed that the binary lens equation can be approximated by a single lens with external shear, or Chang-Refsdal (CR) lens (Chang & Refsdal 1979, 1984), near the individual masses for widely separated binaries ($d \gg 1$), and near the secondary (least massive) lens when $d \ll 1$. Furthermore, near the center of mass of a close binary, the lens equation is well approximated by a quadrupole lens, and both the quadrupole lens and CR lens can exhibit extremely similar magnifications when the quadrupole moment is equated to the shear (Albrow et al. 2002).

Thus, there can exist multiple degenerate solutions to an observed photometric light curve, even with extremely accurate photometry. However, the astrometric behavior of these degenerate solutions is very similar in both the shape and the overall scale of the astrometric curves, at least for the close/wide degeneracy (Gould & Han 2000). Therefore, this degeneracy should not affect the determination of θ_E using the prediction from the light curve. It is likely that the other intrinsic degeneracies will also not affect the determination of θ_E, since the degeneracy arises from the lens equation itself and thus affects both the photometric and astrometric curves in the same manner.

Note that it is important that the normalization of θ_E be consistent for the two degenerate solutions. For example, consider the case of the close/wide degeneracy in MACHO 98-SMC-1 (Afonso et al. 2000; Gould & Han 2000). If one normalizes to the total mass of the binary, the close solution implies a value of $\theta_{E,c} = 76$ μas, whereas the wide solution has $\theta_{E,w} = 170$ μas. Since the astrometric curves are essentially identical (in both shape and scale), one might therefore suspect that the inversion of this process would yield two equally likely values of θ_* that differed by a factor of $\sim$2. Of course, this "ambiguity" is wholly artificial and arises because the value of $\theta_{E,w}$ for the wide binary is normalized to the entire mass of the binary, whereas the lensing effects are basically caused by the least massive lens, since $d = 3.25$. Normalizing to the mass of the single lens, $\theta'_{E,w} = \theta_{E,w}(1 + q^{-1})^{-1/2}$, where $q = 0.24$, and thus $\theta'_{E,w} = 75$ μas, essentially identical to the close binary solution. Note that as d approaches unity, the identification of the "proper" θ_E normalization becomes more nebulous, since the lenses can no longer be considered independent. However, the degeneracies also become less severe as $d \to 1$.

Dominik (1999b) has also shown that poorly sampled binary lens light curves can also yield distinct degenerate solutions. Note that these solutions are "accidental" in the sense that they do not arise from degeneracies in the lens equation itself. Thus, Han et al. (1999) found that such degenerate photometric light curves yield astrometric curves that are widely different. Such degeneracies would prohibit the measurement of θ_E using a few astrometric measurements. Therefore, well-sampled photometric light curves are essential for reliable measurements of θ_*.

4.2.2. *Parallax*

If the two observers are displaced by a significant fraction of $\tilde{r}_E \equiv D\theta_E$, the angular Einstein ring radius projected onto the observer plane, then the source position $\boldsymbol{u}$ as seen by the two observers will be significantly different. Since *SIM* will be in an Earth-trailing orbit, it will drift away from the Earth at a rate of $\sim$0.1 AU yr^{-1}. Thus, after 2.5 yr (halfway through the *SIM* mission), it will be displaced from the Earth by $\ell \sim 0.25$ AU, which corresponds to a displacement in the Einstein ring of

$$|\delta \boldsymbol{u}| = \frac{\ell}{\tilde{r}_E} |\sin \gamma| \,, \tag{10}$$

where γ is the angle between the line of sight and the Earth-*SIM* vector. For typical bulge parameters,

$$\tilde{r}_E = 7.6 \text{ AU} \left(\frac{M}{0.3\ M_\odot} \right)^{-1/2} , \tag{11}$$

and therefore $|\delta \boldsymbol{u}| \sim 3\%$. This implies that both the magnification A and the centroid shift $\delta \boldsymbol{\theta}_{cl}$ will be significantly different (at a given time) as seen from the Earth and *SIM*. Since the value of $\tilde{r}_E$ is not known a priori, $|\delta \boldsymbol{u}|$ cannot be predicted from the ground-based photometry alone and must be estimated from the astrometric data itself. Fortunately, *SIM* will likely have excellent photometric capabilities (see Gould & Salim 1999), and thus the relative magnifications between the light curves from the ground and *SIM* will provide additional constraints. Indeed, in our Monte Carlo simulations, we assumed that the astrometric observer was displaced by 0.2 AU from the Earth, simultaneously fitted for both θ_E and $\tilde{r}_E$ (among other parameters), and found that θ_E could still be constrained quite accurately. This is because the information about $\tilde{r}_E$ comes primarily from the photometry, while the information about θ_E comes primarily from the astrometry. Therefore, the two parameters are not degenerate. Note that a "by-product" of these measurements is a determination of the total mass of the binary lens (Gould & Salim 1999; Han & Kim 2000; Graff & Gould 2002).

There are two additional parallax effects. One is due to the motion of the Earth (or *SIM*) around the Sun and will become significant on timescales that are a substantial fraction of a year, which corresponds to many t_E for typical bulge events. There is also a second-order effect that arises from the difference in projected velocities between the Earth and *SIM*. This effect is $\propto v_\oplus/\tilde{v}$, where $\tilde{v} \equiv \tilde{r}_E/t_E$ is the transverse velocity of the lens projected on the observer plane and is $\sim$800 km s^{-1} for typical bulge self-lensing events. Since the velocities and positions of the Earth and *SIM* will be known, both of these effects can easily be included in the fit for the microlensing parallax and so do not present any additional difficulties.

4.2.3. *Luminous Lenses and Binary Sources*

With its planned 10 m baseline, *SIM* will have a resolution of $\sim$10 mas, sufficient to resolve the majority of unassociated nearby stars that are blended with the source in ground-based photometry (Han & Kim 1999). Since the photometric blending is well constrained in binary lens events, unambiguous prediction of the unblended astrometric behavior of the source is possible. Thus, blending will typically not affect the measurement of θ_E. However, luminous lenses and companions to the source star with separations $\lesssim$10 mas will not be automatically resolved by *SIM* (Jeong, Han, & Park 1999). Dalal & Griest (2001) have shown that, using two pointings, this limit may be lowered to $\sim$3 mas; however, it is essentially impossible to resolve multiple sources with separations below this limit (e.g., binary source companions). In these cases, all that will be measured is the total centroid of all the sources in the resolution element.

The centroid in the presence of luminous lenses, $\boldsymbol{\varphi}_{cl,b}$, is related to the centroid in the absence of blending, $\boldsymbol{\varphi}_{cl}$, by

$$\boldsymbol{\varphi}_{cl,b} = \left(\boldsymbol{\varphi}_{cl} + \frac{f_t}{A} \boldsymbol{\varphi}_b \right) \left(1 + \frac{f_t}{A} \right)^{-1} , \tag{12}$$

where $f_t \equiv \sum_i F_{b,i}/F_0$ is the sum of the flux of all unlensed sources (blends) relative to F_0, the baseline flux of the lensed source, A is the magnification of the source, and $\boldsymbol{\varphi}_b$ is the centroid of the blends relative to the origin of the lens. From equation (12), it is clear that blending is more complicated

in astrometric microlensing than in photometric microlensing: whereas photometric blending can be described by one parameter, the blend fraction f_t, astrometric blending requires two additional parameters, the components of the centroid of light of the blend $\boldsymbol{\varphi}_b$. A special case of astrometric blending is bright-lens blending. In the single-lens case, this eliminates the blend location parameters; the location of the centroid of light is the moving lens (i.e., $\boldsymbol{\varphi}_b = 0$). In bright binary lens blending, only one parameter is eliminated; the centroid of light is somewhere on the lens axis between the two stars in the lens. However, it will generally not be known a priori which case one is dealing with, and therefore $\boldsymbol{\varphi}_b$ must be included as a parameter in the astrometric fit.

Blending is problematic because it effectively "dilutes" the astrometric shift between two points in the light curve, which is qualitatively similar to the effect of changing $\theta_{\rm E}$. If the event is not well covered, these two effects can be quite degenerate. In order to determine how degenerate blending is with the $\theta_{\rm E}$, we have included in our Monte Carlo simulations a fixed amount of blended light of $f_t = 1\%$, 10%, and 90%. We assume that f_t is known (i.e., from photometry), but $\boldsymbol{\varphi}_b$ is not. The results are shown in Figure 2. We find that if the blending fraction is close to 1, then the two effects are nearly degenerate, and our fractional uncertainty in $\theta_{\rm E}$ increases by 2 orders of magnitude to of order unity. However, for $f_t \lesssim 1\%$, the median error increases by less than a factor of 2. In most cases, the blending will be known to be small from the photometric data. In these cases, the fractional uncertainty in $\theta_{\rm E}$ will not be seriously degraded. The few events with known large blending can be easily jettisoned from the sample.

4.2.4. *Lens Rotation*

The photometric effects of lens rotation in binary microlensing events has been explored theoretically by Dominik (1998) and has been detected in event MACHO 1997-BUL-41 (Albrow et al. 2000). The astrometric effects of rotating binary lenses have not been explored, and it is therefore difficult to draw any general conclusions as to the importance of this effect. However, to the extent that it is detectable in the photometric light curve, lens rotation poses no difficulties, as its astrometric effect should be predictable from the global solution. Effects that are photometrically undetectable but astrometrically significant are potentially problematic.

The amount that a binary lens rotates during $t_{\rm E}$ is given by

$$\psi \simeq 4\overset{\circ}{.}5 d^{-3/2}\left(\frac{M}{0.3\ M_\odot}\right)^{1/4}\left(\frac{D'}{1.5\ \rm kpc}\right)^{-1/4} \times \left(\frac{v}{150\ \rm km\ s^{-1}}\right)^{-1}, \tag{13}$$

where $D' = D_{\rm ol}D_{\rm ls}/D_{\rm os}$ and assuming circular, face-on orbits. Because the caustic cross section is maximized for binaries with separations of order $\theta_{\rm E}$, the majority of detected caustic crossings will have $d \sim 1$ (Baltz & Gondolo 2001). Therefore, for typical events, the effects of binary rotation should be small if astrometric observations are closely spaced with respect to the event timescale $t_{\rm E}$. This is generally also advantageous for the accurate recovery of $\theta_{\rm E}$ (see § 3.3).

4.3. *Measuring $D_{\rm os}$*

In this paper, we have emphasized the measurement of the angular radius θ_*, rather than the physical radius R_*. However, it may also be interesting to measure R_* for some events. In order to do this, the distance to the source star must be measured independently. Fortunately, the astrometric accuracy needed to measure θ_* is generally sufficient to measure the parallax π_s of the source stars,

$$\pi_s = 125\ \mu\rm as\left(\frac{D_{\rm os}}{8\ \rm kpc}\right)^{-1}. \tag{14}$$

In order to measure R_* to a similar accuracy as θ_* ($\sim$5%), π_s must be measured to somewhat better accuracy, which implies an astrometric error of $\sigma_\theta \lesssim 5\ \mu$as. For *SIM* and an $I = 18$ source, this is achievable with $\gtrsim 7$ hr of integration, which is considerably more time than is needed for the θ_* measurement alone. However, it is important to note that these measurements can be made after the microlensing event is over. Therefore, it should be possible to employ ground-based interferometers for the measurement of π_s, rather than spend precious *SIM* resources, at least for brighter sources.

4.4. *Typing the Source Star*

A measurement of θ_* is essentially useless if the spectral type and luminosity class of the source is not known. The source stars of microlensing events can be typed in two ways. The first is to simply measure the color and apparent magnitude of the source. This information is generally acquired automatically from the fit to the photometric data of the microlensing event.[9] By positioning the source star on a color-magnitude diagram of other stars in the field, one can generally type the source to reasonable accuracy. The primary pitfalls of this method are differential reddening and projection effects (i.e., the source may be in the foreground or background of the bulk of the stars in the field).

A more robust way of typing the source star is to acquire spectra. This is best done when the source is highly magnified as it crosses a caustic, as this minimizes the effects of blended light and increases the signal-to-noise ratio (S/N). Thus, such measurements require target-of-opportunity observations. For highly magnified events, spectra with S/N $\sim$ 100 per resolution element can be achieved with exposure times of tens of minutes for low-resolution spectra (Lennon et al. 1996) or a couple of hours for high-resolution spectra (Minniti et al. 1998), using 8 or 10 m class telescopes. Although low-resolution spectra are sufficient for accurate spectral typing, high-resolution spectra are desirable for a number of other applications, including resolution of the atmosphere of the source star (Gaudi & Gould 1999; Castro et al. 2001; Albrow et al. 2001b), detailed abundance analysis (Minniti et al. 1998), and detection of a luminous lens (Mao, Reetz, & Lennon 1998). Note also that as a by-product, true space velocities of a sample of stars in the bulge will be obtained by combining the proper motions and parallaxes of the sources acquired from astrometric measurements with radial velocities determined from the spectra.

[9] Note that blending is generally not a problem, as binary lens events can typically be easily deblended.

4.5. *An Example Campaign*

In this section, we review the requirements for measuring θ_* for the source stars of Galactic bulge microlensing events and outline the resources needed for a campaign aimed at measuring θ_* for a significant number of sources.

The first requirement is a large sample of caustic-crossing binary lens events from which to choose targets, which in turn requires an even larger sample of microlensing events. A large sample is important in that it ensures that only interesting and promising sources and events are followed. Currently, both the OGLE and MOA collaborations monitor many millions of stars in the Galactic bulge. Both reduce their data in real time, enabling them to issue "alerts," notification of ongoing microlensing events (Udalski et al. 1994; Bond et al. 2002a).[10] Combined, these two collaborations should alert about 500 events yr^{-1} (with the majority of alerts from OGLE). Extrapolating from previous results (Alcock et al. 2000a; Udalski et al. 2000), approximately 5% of these will be caustic-crossing binaries, or 25 events yr^{-1}. Figure 3 shows the cumulative distribution of apparent (i.e., uncorrected for reddening) I magnitudes of the 438 independent bulge microlensing alerts in 2002 for which baseline magnitudes were available. Of these events, 382 (87%) were alerted by OGLE, 61 (14%) were alerted by MOA, and five were alerted by both collaborations. For the typical colors of sources in the bulge ($V-I \sim 2$), only about 50% of the alerts have $V < 20$ and thus would have been accessible to *SIM*. The boundary between dwarfs and giants will occur at an apparent magnitude that depends on the color of the source, the distance to the source, and the reddening. However, for definiteness we simply assume that the boundary between giants and main-sequence stars occurs at roughly $I = 17$. With this assumption, we can therefore expect that approximately 20% of all alerted events will be due to giant stars. Therefore, we can expect approximately $20\% \times 5\% \sim 1\%$ of all alerts, or roughly five events (assuming 500 alerts), to be caustic-crossing events with giant sources, and $80\% \times 5\% \sim 4\%$, or $\sim$20 events, to be caustic-crossing events with main-sequence sources, roughly eight of which will be bright enough to monitor with *SIM*.[11] These numbers are likely to remain valid at least for the next several years. However, in the more distant future, and in particular by the time *SIM* is launched, it is likely that the next generation of microlensing survey collaborations will have come on line. Thus, we can expect that when *SIM* time is operational,[12] a considerably larger sample of caustic-crossing events will be available.

Survey-type experiments are needed to discover microlensing events toward the bulge, and survey-quality data are generally sufficient to uncover the caustic-crossing nature of the target events. However, as we discussed in § 3.1, more accurate and densely sampled photometry is generally needed during the caustic crossing in order to measure Δt. Currently, there are several collaborations with dedicated (or substantial) access to 1–2 m class telescopes distributed throughout the southern hemisphere that closely monitor alerted microlensing events with the goal of discovering deviations from the single-lens form, with emphasis on the search for extrasolar planets (Albrow et al. 1998; Rhie et al. 2000; Tsapras et al. 2001; Bond et al. 2002b). These collaborations have also been quite successful in predicting and monitoring binary lens caustic crossings. It seems likely that these collaborations, or similar ones, will still be in place when the next generation of interferometers, or even *SIM*, comes on line.

In our Monte Carlo simulations we derived the expected precisions $\sigma_{\theta_\mathrm{E}}/\theta_\mathrm{E}$, assuming that the photometric errors were dominated by photon statistics and that a total of $N = 60,000$ photons were collected over the entire exposure time for each event. This corresponds to total exposure time of $T = 1.6$ hr for *SIM* on an $I = 18$ source. We assume that the fractional photometric uncertainty is $N^{-1/2}$ and that the astrometric uncertainty is related to the photometric uncertainty via the expression, $\sigma_\theta = N^{-1/2}\theta_f$, with $\theta_f = 2.5$ mas, as appropriate for *SIM*. Since we assumed that the photometric and astrometric errors are given simply by photon statistics, it is trivial to scale our results for other total exposure times T and source brightnesses assuming the characteristics of *SIM*: $\sigma_{\theta_\mathrm{E}}/\theta_\mathrm{E} \propto T^{-1/2}$ and $\sigma_{\theta_\mathrm{E}}/\theta_\mathrm{E} \propto 10^{0.2(I-18)}$. For the purposes of planning observations and providing an order-of-magnitude estimate for the number of angular radii that can be measured for a given amount of *SIM* time, it is useful to derive an expression for the exposure time required to achieve a given median photometric precision. To be conservative, we assume that the blending is small, but nonnegligible. Specifically, we adopt the median error

[10] For on-line alerts, see http://www.astrouw.edu.pl/~ftp/ogle/ogle3/ews/ews.html (OGLE) and http://www.roe.ac.uk/~iab/alert/alert.html (MOA).

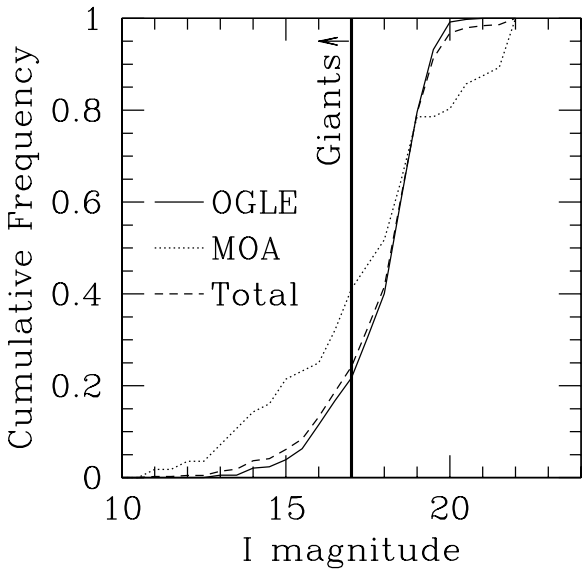

FIG. 3.—Cumulative distribution of I magnitudes for OGLE and MOA alerts in 2002. The solid line is for OGLE, the dotted line is for MOA, and the dashed line is for the total. The majority ($\sim$90%) of alerts are from OGLE.

[11] This assumes that the detection efficiency of binary lens events does not depend on the I magnitude of the source. In fact, deviations from the single-lens form will generally be easier to detect in brighter sources; however, this bias is likely to be relatively small for caustic-crossing events, which generally exhibit dramatic and easily detectable deviations from the point lens form. This may seem in contradiction with the fact that none of the five events toward the bulge presented in Table 1 are main-sequence sources. However, this is almost certainly a selection effect: bright binary lens events are currently preferentially monitored by the follow-up collaborations, in order to achieve a higher signal-to-noise ratio during the second caustic crossing.

[12] The projected launch date for *SIM* is currently 2009.

found for the Monte Carlo simulations assuming a blend fraction of $f_t = 1\%$, which is $\sigma_{\theta_E}/\theta_E \simeq 5.5\%$. Then,

$$T \sim 2\ \mathrm{hr}\left(\frac{\sigma_{\theta_E}/\theta_E}{5\%}\right)^{-2} 10^{0.4(I-18)}\ . \qquad (15)$$

Thus, for giant sources ($I \simeq 15$), 7.4 minutes are required to achieve 5% precision, whereas for main-sequence sources ($I \simeq 20$), 12.3 hr are required for 5% precision, or 1.4 hr for 15% precision.

We have focused here primarily on astrometric observations with *SIM* because its capabilities are well suited to this application. The basic requirements to be able to measure θ_E accurately for the events we have discussed are relatively high astrometric precisions, $\sim$10 μas, and high sensitivity (via, i.e., large apertures), as the sources we are considering are faint, $I = 15$–20. These faint sources are inaccessible to current ground-based interferometers. Upcoming large-aperture, ground-based interferometers, such as the Very Large Telescope Interferometer or the Keck Interferometer, should be able to achieve the requisite astrometric precisions on all of the bright ($I \lesssim 15$) giant events. If the target microlensing source happens to have a bright star within the isoplanatic angle, it may be possible to employ phase referencing to extend sensitivity to very faint ($I \lesssim 20$) sources. This would allow one to measure θ_E for main-sequence sources from the ground as well. Finally, it may be possible to determine θ_E from single-epoch measurements of the visibility and/or closure phase (Delplancke, Górski, & Richichi 2001; Dalal & Lane 2002). In this way, sensitivity could plausibly be extended to main-sequence sources by making carefully timed interferometric measurements of the source when it is highly magnified during a caustic crossing. However, it is not clear if there exists enough structure in the image positions during this time to extract θ_E. This remains an interesting topic for future study. Nontargeted, space-based astrometric surveys, such as the *Global Astrometric Interferometer for Astrophysics*, are generally not well suited to this application, because of the relatively sparse sampling of the source stars.

Finally, access to target-of-opportunity time on 8–10 m class telescopes would allow for accurate spectral typing of the source stars. Several nights per bulge season would likely be adequate to type the $\sim$25 caustic-crossing events yr^{-1}. However, more time would be required to perform some of the auxiliary science discussed in § 4.3, such as resolution of the source star atmospheres.

Thus, by combining alerts from survey collaborations, with comprehensive ground-based photometry from follow-up collaborations having access to dedicated (or semi-dedicated) 1 m class telescopes, and a modest allocation of a total of 10 hr of *SIM* time, it should be possible to measure the angular radii of $\sim$80 giant stars in the bulge to 5%, or roughly seven main-sequence stars to 15%. Several nights of target-of-opportunity time on 8–10 m telescopes should allow for accurate spectral typing of the sources via high- or low-resolution spectroscopy.

5. CONCLUSION

We have outlined a method to measure the angular radii θ_* of giant and main-sequence source stars of fold caustic–crossing binary microlensing events toward the Galactic bulge. Our method to measure θ_* consists of four steps. First, survey-quality data can be used to discover and alert caustic-crossing binary lensing events. Such data are sufficient to characterize the event timescale t_E and the angle ϕ of source trajectory with respect to the caustic. Dense sampling of one of the caustic crossings yields the caustic-crossing timescale Δt. The global solution to the binary lens light curve yields a prediction for the trajectory of the centroid of the source up to an unknown angle α and the scale θ_E. Thus a few precise astrometric measurements during the course of the event yield θ_E. The angular source radius is then simply given by $\theta_* = \theta_E(\Delta t/t_E)\sin\phi$.

We argued, based on past experience with modeling binary lens events, that the parameters Δt, ϕ, and t_E should be measurable to a few percent accuracy, provided that one caustic crossing is densely and accurately sampled and the entire event is reasonably well covered.

We then performed a series of Monte Carlo experiments that demonstrated that astrometric measurements during the course of the binary lens event should allow for the determination of θ_E to $\sim$2% accuracy, assuming photon-limited statistics and a total of 60,000 photons per event. This is a factor of $\sim$50 fewer photons than are required to measure θ_E to the same precision in single-lens events and corresponds to an exposure time of $T = 1.6$ hr with *SIM* on an $I = 18$ source. Therefore, it should be possible to measure θ_* for a significant sample of giant and main-sequence stars in the bulge with reasonable expenditure of resources.

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