Lecture 7: Stellar Radii and Masses
Sections: 19.6, 19.9-11

Stellar Radii

Key Ideas
Almost all stars can be considered "point sources".

> "Size" of stars in images is an artifact of blurring by atmosphere and telescope optics

For the small number of stars which can be seen as bigger than a point, techniques for measuring radii include
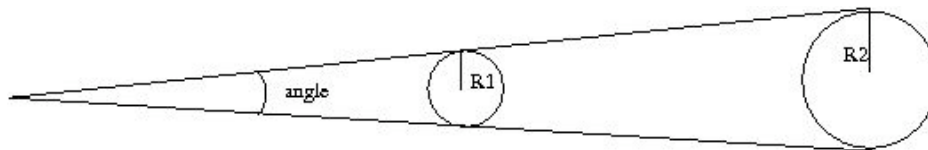
> Direct measurement
> Lunar occultation

For other stars, we need to infer the radii from the temperatures and luminosities.

Radii and Distance

All measurements of radii depend on distances. The figure below shows that the same angular size can mean very different radii, depending on how far we are from the star.

How big are stars as seen from Earth?

Not big (with the exception of the Sun). If we put the Sun at a distance of 10 pc, it would have an angular size of 0.00038 arcseconds, or the size of a dime at 5,400 km, which is about the radius of the Earth. Stars that are further away have even smaller angular sizes. While telescopes magnify the sky, there are limitations to our ability to resolve objects that tiny on the sky.

You *cannot* measure radii of stars from their widths on an image.

Seeing

One of the main problems is that the light from the star has to come through the Earth's atmosphere. In a similar fashion to the way hot air rising from a road or parking lot in the summer causes the objects behind it to look blurry and fuzzy, the atmosphere distorts the light in a constantly changing fashion. When we take an exposure lasting more than a fraction of a second with our cameras, all of these images combine to give us a blob that is about 1 arcsec across.
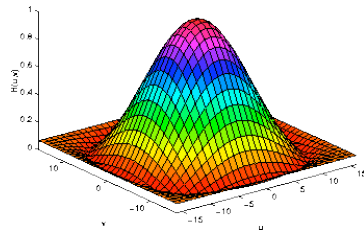
Stars of all brightnesses in an image are blobby. However, we are sensitive to the wings of the blob in the case of stars that are brighter and therefore they appear larger on the image, although they may not be in reality!

Bottom line:
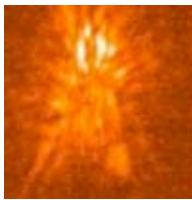
This →

dancing all over the place leads to a big blob forming by the time your exposure is complete. This blob in 3-dimensions looks like

While getting above the Earth's atmosphere helps a lot, there is a fundamental limit to how well a telescope can resolve an object, and therefore direct measurements using angular sizes of stars is limited to just a few.
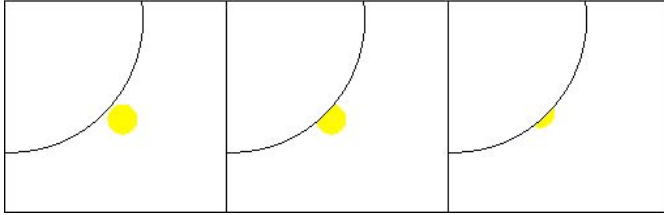
Direct Measurement

The Sun!

Betelgeuse!

That's about it!

Lunar Occultation

This is a cool way of getting around the blurring problem.

The bigger the star, the longer it takes for the Moon to cover it.

Method

      Measure the time it takes for the Moon to pass in front of a star

      Calculate how fast the Moon is moving in arcsec/sec

      Then use the formula:

$$\text{angular diameter} = \text{velocity} \times \text{time}$$

Limitations:

      Moon does not cover all the stars in the sky.

      Moon's angular speed is actually quite large

Calculating the Moon's angular speed:

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{1{,}296{,}000''}{29\,\text{days}} = 0.52\,\frac{\text{arcsec}}{\text{second}}$$

If the Moon is occulting a sun-like star at 10 pc → angular diameter of 0.00038 arcseconds, how long does the occultation event last?

$$t = \frac{d}{v} = \frac{0.00038''}{0.52''/\text{sec}} = 0.0007\,\text{sec}$$

Yikes! We need a camera that can take very fast exposures and a star that is bright enough that we can detect it with such short exposures. So there have around 150 stars with lunar occultation measurements, which is helpful but still limited..

Radii from Temperatures and Luminosities

In the first week, we saw that stars had spectra very similar to blackbodies, close enough that we can use equations for blackbodies to learn about the stars.

Stefan-Boltzmann Law gives us the energy radiated per sec per unit area on the surface of a blackbody

$$E = \sigma T^4$$

where σ is a constant.

The total amount of energy radiated per second by a star (the luminosity) can be found by multiplying E in the equation above by the total surface area of the star. For a sphere, geometry tells us that the area is $4\pi R^2$, where R is the radius of the star.

$$L = 4\pi R^2 \sigma T^4$$

This is a very handy equation, because, regardless of the angular size of the star, we are still getting photons from it and can calculate the luminosity. We can also find the temperature using Wien's law or other techniques. Therefore, we can then find the radius using

$$R = \sqrt{\frac{L}{4\pi\sigma T^4}}$$

Example: the Sun

Summary

Difficult to measure because stars are so far away.
Radii have been measured for ~600 stars
Stars come in all sizes!

Stellar Masses

Key Ideas

Measure stellar masses from binary stars
        Only way to measure stellar masses
        Only measured for ~150 stars

Types of Binary Stars
        Visual
        Eclipsing
        Spectroscopic

Measuring Masses

Masses are measured by using the effects of gravity on objects:
        Your mass from how much the Earth's gravity pulls on you ("weight")
        Earth's mass from the orbital motions of the Moon or artificial satellites
        Sun's mass from the orbital motions of the planets

Binary Stars
<u>Apparent Binary Stars</u> (that is, Fakes)
        Chance projection of two distinct stars along the line of sight
        Often at very different distances
<u>True Binary Stars</u>
        A pair of stars bound by gravity
        Orbit about their common center of mass
        About 60% of all systems have 2 or more stars

Types of True Binary Stars
        These types are defined by how we view them from Earth. They are not an intrinsic property of the stars

Visual Binary
        See both stars and follow their orbits over time
Spectroscopic Binary

Stars are too close to see as separate stars, but we detect their orbital motions by the Doppler shifts of their spectral lines.
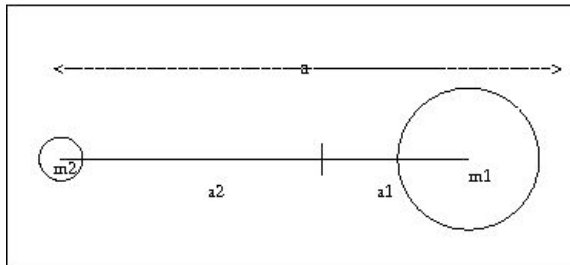
Eclipsing Binary

Too close to see as separate stars, but we see the total brightness of the system decrease when they periodically eclipse each other.

Visual Binaries

Center of Mass

Two stars orbit about their common center of mass:

Measure semi-major axis from projected orbit and *distance*

Relative positions give $\dfrac{M_1}{M_2} = \dfrac{a_2}{a_1}$. So now you know the ratios of the masses.

Newton's version of Kepler's Third Law provides additional information

$$P^2 = \frac{4\pi^2}{G(M_1 + M_2)} a^3$$

Measure Period, P, by following the orbit

Measure semi-major axis, a, from the angular separation on the sky and the distance

Solve for the total mass $(M_1 + M_2)$

Estimate mass ratio (M_1/M_2) from the projected orbit, then solve for the individual masses

Problems

We need to follow an orbit long enough to trace it out in detail

This can take decades

Need to work out the projection on the sky

Measurements depend on the distance

Semi-major axis

Derived mass depends on d^3
Small distance errors add up quickly!

Spectroscopic Binaries

Most binaries are too far away to see both stars separately
But you can detect their orbital motions by the periodic Doppler shifts of their spectral lines.
Determine the orbit period and size from velocities

Problems
Often don't see the two stars separately
Semi-major axis must be estimated from the orbital parameters
Can't tell how the orbit is tilted on the sky

Everything depends critically on knowing the inclination.

Eclipsing Binaries
Two stars orbiting nearly edge-on
See a periodic decrease in the total brightness of the system as one star eclipses the other.
Combine with spectra to measure the orbital speeds with time.
With the best data, one can find the masses *without* having to know the distance!

Problems
Eclipsing Binaries are very rare.
Measurement of the eclipse light curves complicated by details
Practical eclipses yield less accurate masses
Atmospheres of the stars soften edges
Close binaries can be tidally distorted
Best masses are from eclipsing binaries.

Stellar Masses
Masses are known for only ~150 stars.
Range: about 0.07 to 60 solar masses
Rare stars with masses possibly as high as 80-120 solar masses

Stellar masses can only be measured for stars with a companion.