Astronomy 350, Autumn 2002, Problem Set 5

Due Thursday, November 7 in class

Problem 1

The thermal conductivity, Q, of a cylindrical bar of copper is proportional to its cross-sectional area, $A=\pi r^2$, where r=radius of the bar, and inversely proportional to its length, L:

a) What is the fractional uncertainty of the derived thermal conductivity, σ_Q/Q, expressed in terms of the fractional uncertainties of the measured radius and length of the bar?

b) Which quantity, L or r, should be measured with greater precision to better determine Q? Please be quantitative (e.g., how much better)?

This kind of problem is an example of "working out the error budget" for a system.

Problem 2

You have 10 images of a faint planetary nebula , and for each you have measured the apparent angular diameter, D, in arcseconds, and its uncertainty, σ_D as follows:

D (arcsec)	24.3	25.4	24.7	27.9	22.4	21.0	22.2	23.5	23.7	22.0
σ_D (arcsec)	0.5	0.6	0.5	0.8	0.3	0.2	0.3	0.4	0.5	0.3

a) Compute the weighted mean and its uncertainty from the data above.

b) Compute the *unweighted* mean and its uncertainty. How does it compare to the weighted mean? What do you think is behind the difference?

Problem 3

Fill in the right side of the table below with the error propagation formulae for common functional forms *f(x,y)*. Here, x and y are "observables" with uncertainties σ_x and σ_y, respectively, and a,b, etc. are constants with *no uncertainties*. In addition to computing σ_f, *if it is practical* also derive the fractional uncertainty (σ_f/f) in terms of the fractional uncertainties of x and y.

Functional Form	Error Formula
$f = ax \pm by$	$\sigma_f =$
$f = \pm axy$	$\sigma_f =$
$f = \pm ax / y$	$\sigma_f =$
$f = \pm ax^{\pm b}$	$\sigma_f =$
$f = a \ln(\pm bx)$	$\sigma_f =$
$f = x \pm a \log_{10}(y)$	$\sigma_f =$

[Please make your own table, don't just fill in the blank example above, you will need more space.]

Problem 4

The "activity" of a radioactive source is measured in terms of the number of decays observed per second. Over time, the activity decreases exponentially as

$$N(t) = N_0 e^{-t/\tau}$$

where N_0 is the "initial activity" at time t=0, and τ is the "lifetime" of the radioactive source. Suppose that for a particular experiment, both N_0 and τ are both known to a precision of 1%.

a) Show that the uncertainty in the activity at a given time, $\sigma_N(t)$, is dominated by the uncertainty in the initial activity σ_{N0} for early times t, and by the uncertainty in the lifetime σ_τ at later times t.

b) At what value of (t/τ) do the uncertainties in N_0 and τ contribute equally to the uncertainty in the observed activity, σ_N?

Problem 5

The observed counts from an astronomical source are converted into a magnitude using a formula like the following:

$$m = m_0 - 2.5\log_{10}(C)$$

where C is the observed counts, and m_0 is the "Zero Point" of the photometric system.

a) Suppose we know the zero point to perfect precision, and the observed uncertainty on the counts is σ_C. What is the uncertainty on the derived magnitude, σ_m?

b) Suppose that the zero-point has an uncertainty σ_{m0}, and the uncertainty on the observed counts is still σ_C. What is σ_m now?