Dealing with Rayleighs and Emission Measure: a worked example

In ISM studies, the standard unit of surface brightness for emission lines is the Rayleigh:

$$1 \text{ Rayleigh} = \frac{10^6}{4\pi} \text{ photons s}^{-1} \text{ cm}^{-2} \text{ sr}^{-1}$$

The conversion between Rayleighs and surface brightness expressed in energy flux per square arcsec (e.g, erg s^{-1} cm^{-2} $arcsec^{-2}$) depends on the wavelength of the emission line:

To convert, we need some useful numbers:

1 photon = hν or hc/λ ergs = $1.98645\times10^{-8}/\lambda_{\text{Å}}$ ergs
1 steradian = $32400/\pi^2$ $degrees^2$ = 4.254517×10^{10} $arcsec^2$

So

$$1 \text{ Rayleigh} = 1.8704\times10^{-6} \text{ hν erg s}^{-1} \text{ cm}^{-2} \text{ arcsec}^{-2}$$

or

$$1 \text{ Rayleigh} = 3.71546\times10^{-14} \lambda^{-1} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ arcsec}^{-2}$$

with λ in Angstroms.

For different recombination lines:

Hα: λ=6562.80Å
Hβ: λ=4861.32Å
Lyα: λ=1215.67Å

Thus

Hα: 1 Rayleigh = 5.661×10^{-18} erg s^{-1} cm^{-2} $arcsec^{-2}$
Hβ: 1 Rayleigh = 7.643×10^{-18} erg s^{-1} cm^{-2} $arcsec^{-2}$
Lyα: 1 Rayleigh = 3.056×10^{-17} erg s^{-1} cm^{-2} $arcsec^{-2}$

The Emission Measure (EM), for pure recombination can be written:

$$EM = 2.75T_4^{0.9} I(H\alpha) \text{ cm}^{-6} \text{ pc}$$
$$= 7.84T_4^{0.9} I(H\beta) \text{ cm}^{-6} \text{ pc}$$

for surface brightness, I(line) expressed in units of Rayleighs, or

$$EM = 4.858\times10^{17} T_4^{0.9} S(H\alpha) \text{ cm}^{-6} \text{ pc}$$
$$= 1.026\times10^{18} T_4^{0.9} S(H\beta) \text{ cm}^{-6} \text{ pc}$$

for surface brightness, S(line) expressed in units of erg s^{-1} cm^{-2} $arcsec^{-2}$.

For Lyα, the situation is a little more complicated. If we adopt the Case A ratio of Lyα/Hβ≈33, this implies

$$EM \approx 0.24T_4^{0.9} I(Ly\alpha) \text{ cm}^{-6} \text{ pc}$$