Astronomy 871, Au2008, Problem Set 2

Due Wednesday, October 15 in class

The interstellar Deuterium abundance (D/H) near the Sun is in the range $D/H \approx 1.2-1.7\times10^{-5}$. Observations of the fictitious "Alice's Star" with EUVE satellite gave an estimate of the neutral Hydrogen column density towards this star of 5×10^{18} cm^{-2}, and the velocity dispersion of HI in this direction is b=10 km s^{-1}. This problem is a proof-of-concept calculation to see if you could use a Far-UV spectrometer on a proposed (also fictitious) UVSpec satellite to measure D/H in the ISM towards Alice's Star. This problem will assume that Alice's Star is bright enough that signal-to-noise is not an issue, and so what you have to assess is whether or not the HI Lyβ line is so saturated as to obliterate the nearby weaker DI Lyβ line.

a) Compute γ_u, b_λ, and $a = \gamma_u / b_\lambda$ for the HI Lyβ line (1s-3p transition). Use transition probabilities and wavelengths for HI lines listed below. Given that the atomic transition probabilities are approximately the same for DI and HI, what are b_λ and a for DI Lyβ?

b) Estimate the optical depths at line center (τ_0) for HI and DI Lyβ, assuming the observed column density of HI towards Alice's Star given above and the range of D/H expected.

c) On what part of the Curve of Growth are the HI and DI Lyβ lines, respectively? Be quantitative in your answers (i.e., don't just assert it, demonstrate it). Is it possible to estimate the HI column density towards Alice's star using this line?

d) Using your results for parts (b) and (c), estimate the expected equivalent widths of the HI and DI Lyβ lines, using the approximation formula for W_λ appropriate to the part of the C-o-G that each line resides upon. How much does W_λ for the DI line change at the extremes of the expected range of D/H?

e) Compute and plot synthetic spectra of the combined HI and DI Lyβ absorption lines for Alice's Star. For each line you need to compute

$$\tau_\lambda = \tau_0 H(a,u) \approx \tau_0 \left[H_0(u) + a H_1(u) \right]$$

where $H_0(u)$ and $H_1(u)$ are the first two terms of the Hjerting function given in the notes. For your synthetic spectra, normalize the continuum flux to 1.0, and assume that each UVSpec spectral pixel is 0.015Å wide. Make two synthetic spectra that bracket the expected range of D/H in the nearby ISM. You are permitted to ignore noise and instrumental broadening in your model spectra. Show plots of your spectra, labeling all axes and lines (use SM).

f) Looking at your synthetic spectra, could you detect the DI Lyβ line in UVSpec data for Alice's Star? Can you tell the difference between the extreme values of D/H expected from other observations? Be as quantitative in your arguments as possible.

Relevant HI Transisions (from the NIST Atomic Spectra Database http://physics.nist.gov/cgi-bin/AtData/lines_form).

Line	Transition	λ_0 [Å]	A_{ul} [s^{-1}]	g_u	g_l
Lyα	2p–1s	1215.67	6.265×10^8	6	2
Lyβ	3p–1s	1025.72	1.672×10^8	6	2
Hα	3p-2s	6562.75	2.245×10^7	6	2