

# Astronomy 871, Autumn 2008, Problem Set 3

Due Thursday, October 30

## Problem 1

A “radiation-bounded” nebula is one in which all of the ionizing photons are absorbed within the ionized volume, and the ionization equilibrium is found by balancing the total number of ionizations within the volume by the total number of recombinations. The radius of a Strömngren Sphere around a star for a given ionic species  $X^i$  is found by solving the equation of ionization equilibrium for that ion:

$$Q(X^i) = \int_{\nu_{0,i}}^{\infty} \frac{L_{\nu}}{h\nu} d\nu = n_e n_{X^i} \alpha(X^i, T) \times \frac{4\pi}{3} r_i^3$$

where  $\nu_{0,i}$  is the ionization threshold frequency for that ion, and  $\alpha(X^i, T)$  is the total recombination coefficient for the ionic species (with units of  $\text{cm}^3 \text{s}^{-1}$ ). For this problem, consider a homogeneous region of space filled with only H and C with  $C/H=4 \times 10^{-4}$  (i.e., no He or anything else). The values of the ionization cross-sections relevant to this problem are:

$$a_{\nu}(H^0) = 6.30 \times 10^{-18} \left( \frac{\nu}{\nu_0} \right)^{-3}$$

$$a_{\nu}(C^0) = 1.22 \times 10^{-17} \left( \frac{\nu}{\nu_0} \right)^{-2}$$

good near the ionization thresholds,  $h\nu_0=13.6\text{eV}$  and  $h\nu_0=11.2\text{eV}$ , for  $H^0$  and  $C^0$ , respectively.

- a) Consider this statement: “It is a reasonable approximation to assume that all ionizing photons with energies  $h\nu > 13.6\text{eV}$  ionize only  $H^0$ , while all photons with energies of  $11.2 < h\nu < 13.6\text{eV}$  only ionize  $C^0$ .”. Is this a “reasonable approximation”? Why? (make a quantitative argument)
- b) Assume that the statement above is valid, and compute the relative number of photons that can ionize  $H^0$  and  $C^0$ ,  $Q(C^0)/Q(H^0)$ , from a B5v star. Idealize the star as a  $T=15000\text{K}$  blackbody using the Wien approximation:

$$B_{\nu} \approx \frac{2h\nu^3}{c^2} e^{-h\nu/kT}$$

to represent the stellar ionizing continuum spectrum. You can evaluate the integral numerically or analytically as you wish.

- c) Compute the relative radii of the  $H^+$  and  $C^+$  Strömngren spheres around this idealized star. For this calculation, assume that all the electrons in the  $H^+$  zone come from  $H^0$ , and that all the electrons in the  $C^+$  zone come from  $C^0$ . The approximate total recombination coefficients to use are

$$\alpha(H^0, T) = 2.59 \times 10^{-13} \text{ cm}^3 \text{ s}^{-1}$$

$$\alpha(C^0, T) = 4.66 \times 10^{-13} \text{ cm}^3 \text{ s}^{-1}$$

- d) For typical ISM conditions, the mean hydrogen density is  $0.3 \text{ cm}^{-3}$ , and the radius of an  $H^+$  Strömngren sphere around a B5v star is  $r_H \approx 4 \text{ pc}$ . What is the radius of this star’s  $C^+$  Strömngren sphere? Comment on the value you get.
- e) In part d, we assumed that all of the electrons in the diffuse ISM come from  $C^0$  ionization, but we know that this isn’t really true. How much does adopting the typical interstellar electron density of  $\langle n_e \rangle \approx 0.03 \text{ cm}^{-3}$  change your answer in part d?

**Problem 2:**

This problem is meant to help you build your intuition about the optical depth in the ISM to various radiative processes. Cross-sections for various processes that you need are as follows:

Thompson cross-section for electron scattering:  $\sigma_e = 6.65 \times 10^{-24} \text{ cm}^2$  (non-relativistic)

Photoionization cross-section for  $\text{H}^0$  at threshold:  $\sigma_{\text{PI}}(\text{H}^0) = 6.30 \times 10^{-18} \text{ cm}^2$

HI Ly $\alpha$  absorption cross-section at line center:  $\sigma_{\text{Ly}\alpha} = 1.215 \times 10^{-13} \left( \frac{b}{\text{km s}^{-1}} \right)^{-1} \text{ cm}^2$

In this notation (cross-section  $\equiv \sigma$ ), the optical depth along a given line of sight is  $\tau = \int_{\text{los}} n\sigma ds$

Consider a sight-line through a mean ISM characterized by electron density  $\langle n_e \rangle \approx 0.03 \text{ cm}^{-3}$ , neutral hydrogen density  $\langle n_{\text{H}}^0 \rangle \approx 0.3 \text{ cm}^{-3}$ , and hydrogen Doppler width  $b \approx 10 \text{ km s}^{-1}$ , what distance through this mean ISM do you have to go before you have one optical depth due to

- a) Electron scattering
- b) Hydrogen photoionization
- c) HI Ly $\alpha$  absorption at line center

Now consider a dense HII region around an O5v star with a density of  $n_{\text{H}}=10^3 \text{ cm}^{-3}$ , a Strömgen radius of  $R_S=1\text{pc}$ , and a neutral fraction of  $4 \times 10^{-4}$  in the interior. What is the optical depth from the star to  $R_S$  due to

- d) Electron scattering
- e) Hydrogen photoionization
- f) HI Ly $\alpha$  absorption at line center (assume purely thermal broadening for  $T \approx 10^4 \text{ K}$ )