Astronomy 871, Autumn 2008, Problem Set 4

Due Wednesday Nov 12

Problem 1:

Using Table II from Panagia 1973 [AJ, 78, 929], and noting that what he calls N_L is our $Q(H^0)$, and the values of the Case B and effective recombination coefficients for H^0 given below, consider ionization-bounded Hii regions with a constant density of $n=100$ cm^{-3} and a temperature of 10^4 K. Compute the following questions for stars of spectral type O4v, O6v, B0v, and B2v:

a) The radii of the Strömgren sphere, r_1, for each star, in parsecs

b) The total luminosity of the Hα emission line, L(Hα) in erg s^{-1}.

In each of parts (a) and (b), first derive the necessary formulae for r_1 and L(Hα), then present a table of your results. The H^0 recombination line data you need are:

$$\alpha_B(H^0,T) = 2.59\times10^{-13}\ \text{cm}^3\ \text{s}^{-1}$$
$$\alpha_{H\beta}^{eff}(H^0,T) = 3.03\times10^{-14}\ \text{cm}^3\ \text{s}^{-1}$$
$$j_{H\alpha} / j_{H\beta} = 2.87$$
$$\lambda_{H\beta} = 486.136\ \text{nm}$$
$$\lambda_{H\alpha} = 656.285\ \text{nm}$$

these values are for $T=10^4$ K from Osterbrock & Ferland (AGN^2 3rd ed).

Problem 2:

You are given the following table of emission-line strengths for the integrated spectrum of a planetary nebula, all measured relative to the flux of Hβ:

Line	I/I(Hβ)
Hγ	0.517
[OIII] 4364Å	0.095
Hβ	1.000
[OIII] 4959Å	4.25
[OIII] 5007Å	12.46
[NII] 5755Å	0.046
[NII] 6548Å	1.04
Hα	2.86
[NII] 6583Å	3.20
[SII] 6716Å	0.222
[SII] 6731Å	0.209

Each of these line strengths has a fractional uncertainty of ~5%. You may also assume that the reddening towards this PNe is negligible. Compute the following, making estimates of the uncertainties as you go:

a) Electron density, n_e, from the [SII] doublet ratio $R_{S^+} = I_{6716} / I_{6732}$

b) Electron Temperature in the O^{++} zone, $T_{O^{++}}$, using the [OIII] line ratio $R_{O^{++}} = (I_{4959} + I_{5007}) / I_{4363}$

c) Electron Temperature in the N^+ zone, T_{N+}, using the [NII] line ratio $R_{N^+} = (I_{6548} + I_{6583}) / I_{5755}$

Use the formulae for the electron temperatures given in the class notes. For the [SII] doublet ratio as a function of ne, use the data from Cai & Pradhan (1993, ApJS, 88, 329) provided on the class website in distilled form as the file named **`ne_sii.txt`**. These data are computed for a range of T_e and n_e.

Make a first pass at computing n_e by making the simplifying assumption that the temperature is about 10^4 K, then compute the temperatures using your derived first-guess n_e. Using the N^+ temperature, re-estimate n_e at the appropriate temperature and iterate to refine your estimates for the O^{++} and N^+ temperatures. As you iterate, ask yourself: "Are the refinements bigger or smaller than my measurement errors?" This is how you know when to quit iterating.

[PS: also, don't kill yourself propagating the errors – reasonable estimates rather than formal estimates are adequate here. Think about ways to estimate the errors without doing a full, explicit error analysis, not least of which because the formulae for temperature have extremely ugly derivatives with respect to T and n_e, and the data for n_e as a function of R_{S+} are numerical computations without a convenient analytic form and you can't use the standard error propagation formulae anyway. I know it's frustrating, but that's what real-world measurements are like more often than not...]