# Astronomy 1143 Homework 1

October 7, 2015

1. Two Martian astronomers, Marvin and Marla, are located due north and south of each other on the planet Mars. Marvin sees the Sun directly overhead (at the zenith) at noon. At the same time, Marla sees the Sun 5 degrees away from the zenith. Marla is 295 kilometers north of Marvin. Using this information, compute the circumference of the planet Mars.

   The distance between our two astronomers is $d_{\text{M-M}} = 295\text{km}$. The Sun angle for Marla is $\theta_{\text{Marla}}$. Now, the difference of angle for Marla at Noon tells us the angular distance between where Marla and Marvin are standing on Mars. Recall that the circumference (total length) of a circle is $C = 2\pi r$, where $r$ here is the total radius of the circle. This is because the angular extent of a circle, in radians, is $2\pi$. If we want to know instead the length of line along the surface of a circle for a smaller angle, which I'll refer to as $s$ here, we have $s = \theta r$, where $\theta$ is of course still in radians. Inverting this equation, we can solve for the Martian radius knowing the length of arc between Marvin and Marla!

$$\begin{aligned}
s &= \theta r && (1)\\
r &= \frac{s}{\theta} && (2)\\
r_{\text{Mars}} &= \frac{d_{\text{M-M}}}{\theta_{\text{Marla}}} && (3)\\
r_{\text{Mars}} &= \frac{295\text{km}}{5 \times \frac{\pi}{180}} && (4)\\
r_{\text{Mars}} &= \frac{295\text{km}}{5 \times \frac{\pi}{180}} && (5)\\
r_{\text{Mars}} &= 3380\text{km} && (6)
\end{aligned}$$

   Note that in going from step 3 to 4, we had to convert the angle we were given from degrees to radians. Since $2\pi$ radians are in one full circle, which is $360°$, we have the conversion as $\frac{\pi \text{ radians}}{180°}$. As a final thing to pay attention to, since our distance was given in kilometers, this is also what our radius is in units of! If we look up the actual radius of Mars, we get 3389.5km, which is very close to what we calculated above!

2. Suppose that Aristarchus had measured an angle of 45 degrees between the Sun and the Moon when the Moon was in its first quarter phase. Draw an accurate diagram of the positions of Earth, Moon, and Sun necessary for this measurement to be correct. In this case, what is the ratio of the Earth-Sun distance to the Earth-Moon distance?

   If the Moon is in quarter phase, this means that half of the Moon is illuminated from the point of view of us here on Earth. This also has the handy attribute of meaning that the angle between the Sun and the Earth from the Moon's point of view is 90°, since the line dividing illuminated and not illuminated sides of the Moon, perpendicular to the Sun's position, is pointing directly at Earth. Now, if the other two sides of the triangle are both 45°, does this make sense? First off, all triangle angles should sum to 180°. 45 + 45 + 90 = 180, so we're fine on that account. Now, what does this mean for the distances? Well, if two angles in a triangle are equal, it means that the side opposite each of them is of equal length. For our geometry, this implies the side connecting the Earth and Moon (opposite the angle at the Sun's position) and the side connecting the Moon and Sun (opposite the angle at the Earth's position) are the same length. Hmmm, so the Moon is as far away from the Sun as the Earth is from the Moon? That seems a little close! For a 45-45-90 degree right triangle, the longest side (the hypotenuse) is as long as the other two sides * $\sqrt{2}$, which is a factor of about 1.4. So is the Sun only 1.4x as far away from the Earth as the Moon is? No, it's much farther! So this observation clearly is incorrect- either the angle was not 45°or the Moon was not in first quarter phase.

3. (a) The Lyman series of hydrogen spectral lines are due to the decay of an electron's orbit from a higher-energy level to which lower-energy level? (i.e. What is $n$?)

   (b) If a photon is emitted by an electron transitioning from an energy level with $E_2 = 10.2$ electronvolts (eV) to an energy level with $E_1 = 0$ eV, what is the energy of the photon emitted, in eV?

   (c) What is the wavelength of the emitted photon in nanometers? Hint: Planck's constant in units of electronvolts-seconds is $h = 4.1 \times 10^{-15}$ eV s.

Remember to keep track of units!

(A) The Lyman series of spectral lines occur when an electron in a hydrogen atom descends to the $n = 1$ state.
(B) Energy is conserved, so $E_{\text{photon}} = E_2 - E_1 = 10.2\text{eV} - 0.0\text{eV} = 10.2\text{eV}$.
(C) The energy of a photon is related to its wavelength by $E = \frac{hc}{\lambda}$. Rearranging for $\lambda$, we have $\lambda = \frac{hc}{E} = \frac{10^{-15}\text{eV*s}(3\times10^{8}\text{m/s})}{10.2\text{eV}} = 2.94 \times 10^{-8}\text{m} = 29\text{nm}$.

4. At its closest approach, the planet Saturn is 8 astronomical units (AU) from the Earth. When Saturn is this close, how long does it take light to travel from Saturn to the Earth? The Suns nearest neighbor among the stars, a dim little star called Proxima Centauri, is 1.295 parsecs (pc) from the Earth. How long does it take light to travel from Proxima Centauri to the Earth?

   If you recall, the Earth is about 8 light-minutes from the Sun. The Earth's average distance from the Sun, by definition, is 1AU. So off the bat, we can know the time it takes for Saturn to get to the Earth in this configuration is about 8 times longer, or 64 minutes (just over an hour). But let's calculate it using the fact that we know light travels at a fixed speed, $c$.

$$\begin{aligned} t_{\text{Saturn}} &= \frac{d}{c} && (7) \\ &= \frac{8\text{AU}}{3 \times 10^8\text{m/s}} \times \frac{1.5 \times 10^{11}\text{m}}{1\text{AU}} \times \frac{1\text{minute}}{60\text{seconds}} && (8) \\ &= 67 \text{ minutes} && (9) \end{aligned}$$

   About what we expected! Now to do the calculation for Proxima Centauri. It's the same thing essentially, except we care about converting parsecs to meters instead of AU to meters. Remembering that 1pc is about 3 light-years, we can guess this will be about 4 years. Let's find out!

$$\begin{aligned} t_{\text{Proxima Centauri}} &= \frac{d}{c} && (10) \\ &= \frac{1.295\text{pc}}{3 \times 10^8\text{m/s}} \times \frac{3.086 \times 10^{16}\text{m}}{1\text{pc}} \times \frac{1\text{year}}{60 \times 60 \times 24 \times 365 \text{ seconds}} && (11) \\ &= 4.2 \text{ years} && (12) \end{aligned}$$

   Again, we were very close!

5. As we have seen in lecture, if the Hubble constant is $H_0 = 71$ km/sec/Mpc, then the Hubble time is $\frac{1}{H_0} = 14$ billion years. Edwin Hubble himself, because he grossly underestimated the distance to galaxies, believed that the Hubble constant was $H_0 = 500$ km/sec/Mpc. For $H_0 = 500$ km/sec/Mpc, what is $\frac{1}{H_0}$, in billions of years?

The Hubble Time, defined in the problem, is given as $t_H = \frac{1}{H_0}$. If you look closely, you'll notice that the Hubble Constant actually has units of inverse seconds, or $\frac{1}{\text{time}}$. This problem is all about keeping track of units! The Hubble Time for Hubble's initial value can be solved for in seconds after converting km to Mpc or vice versa, then seconds into billions of years. Here we go!

$$\begin{aligned}
t_H &= \frac{1}{H_0} && (13)\\
&= \frac{1}{500\text{km/s/Mpc}} && (14)\\
&= 0.002\frac{\text{Mpc*s}}{\text{km}} && (15)\\
&= 0.002\frac{\text{Mpc*s}}{\text{km}} \times \frac{1\text{km}}{1000\text{m}} \times \frac{3.086 \times 10^{22}\text{m}}{1\text{Mpc}} && (16)\\
&= 6.172 \times 10^{22}\text{s} \times \frac{10^9 \text{ years}}{60 \times 60 \times 24 \times 365 \times 10^9\text{s}} && (17)\\
&= 2 \text{ billion years} && (18)
\end{aligned}$$

Now we stop and think if this makes sense. If the Hubble Constant is higher, that means galaxies are expanding very quickly away from one another. This implies that the galaxies can get to their current separations much faster than if they were moving slowly. If they get their faster, then the universe must be younger than we thought, otherwise, they'd be much farther apart by now! So yes, this answer makes sense.