

ASTRONOMY 294Z: The History of the Universe  
Professor Barbara Ryden

## PRACTICE MINI-EXAM

### Short Answer Problems

- 1) *Who lived first: Kepler or Copernicus?*  
Copernicus. (Kepler modified the Copernican model by adding elliptical orbits.)
- 2) *If the density of the universe were greater than the critical density, would the universe be positively curved, negatively curved, or flat?*  
Positively curved.
- 3) *Which is more massive, an electron or a neutrino?*  
An electron.
- 4) *A newly formed zircon crystal contains 1000 uranium-238 atoms. How many uranium-238 atoms will be left after two half-lives?*  
250.
- 5) *A galaxy has a radial velocity of 14,000 kilometers per second. Using Hubble's Law, what is its distance from us?*  
 $d = v/H_0 = 14,000 \text{ km/sec}/70 \text{ km/sec/Mpc} = 200 \text{ Mpc}$
- 6) *Arrange the following objects in order of increasing mass: Jupiter, Sun, brown dwarf, Earth.*  
Earth, Jupiter, brown dwarf, Sun.
- 7) *Which has the longer wavelength, visible light or X-rays?*  
Visible light.
- 8) *If a star is at a distance of 2 parsecs, what is its parallax angle, in arcseconds?*  
 $d = 1/p$ , where  $d$  is the distance in parsecs, and  $p$  is the parallax angle in arcseconds. Thus,  $p = 1/d = 0.5 \text{ arcsec}$ .

## Mathematical Problems

9) *The stars Arcturus and Vega have the same flux as seen from the Earth. Arcturus is at a distance of 11.3 parsecs from the Earth; Vega is at a distance of 7.76 parsecs from the Earth. What is the ratio of the luminosity of Arcturus to the luminosity of Vega?*

The flux of Arcturus,  $F_A$ , is given by the relation

$$F_A = \frac{L_A}{4\pi d_A^2}, \quad (1)$$

where  $L_A$  is the luminosity of Arcturus and  $d_A$  is the distance to Arcturus. Similarly, the flux of Vega is

$$F_V = \frac{L_V}{4\pi d_V^2}, \quad (2)$$

where  $L_V$  is the luminosity of Vega and  $d_V$  is the distance to Vega. Since  $F_A = F_V$ , we can equate the right-hand sides of equations (1) and (2) to find

$$\frac{L_A}{4\pi d_A^2} = \frac{L_V}{4\pi d_V^2}. \quad (3)$$

Canceling the factors of  $4\pi$  and rearranging the equation, we find

$$\frac{L_A}{L_V} = \frac{d_A^2}{d_V^2} = \frac{(11.3 \text{ pc})^2}{(7.76 \text{ pc})^2} = 2.12. \quad (4)$$

In order to appear as bright as Vega, despite being further away, the star Arcturus must have a greater luminosity than Vega.

10) *The star Proxima Centauri has a mass  $0.12M_{\text{sun}}$ , where  $M_{\text{sun}}$  is the Sun's mass. Also, Proxima Centauri has a luminosity  $0.00014L_{\text{sun}}$ , where  $L_{\text{sun}}$  is the Sun's luminosity. If the Sun's lifespan is  $t_{\text{sun}} = 10$  billion years, what is the lifespan of Proxima Centauri?*

The lifespan of a star is directly proportional to its mass (the size of its "fuel tank") and inversely proportional to its luminosity (the rate at which it "burns" fuel). As a mathematical formula, this can be written as

$$t = K \frac{M}{L}, \quad (5)$$

where  $t$  is the lifespan of a star,  $M$  is its mass,  $L$  is its luminosity, and  $K$  is a constant. For the Sun,

$$t_{\text{sun}} = K \frac{M_{\text{sun}}}{L_{\text{sun}}} . \quad (6)$$

For Proxima Centauri,

$$t_{\text{prox}} = K \frac{M_{\text{prox}}}{L_{\text{prox}}} . \quad (7)$$

Dividing equation (7) by equation (6), we find

$$\frac{t_{\text{prox}}}{t_{\text{sun}}} = \frac{M_{\text{prox}}}{M_{\text{sun}}} \frac{L_{\text{sun}}}{L_{\text{prox}}} . \quad (8)$$

(Notice that we don't care what the numerical value of the constant  $K$  is.) We conclude that the lifespan of Proxima Centauri is

$$t_{\text{prox}} = t_{\text{sun}} \frac{M_{\text{prox}}}{M_{\text{sun}}} \frac{L_{\text{sun}}}{L_{\text{prox}}} = 10 \text{ billion years} \times 0.12 \times \frac{1}{0.00014} = 8.6 \text{ trillion years} . \quad (9)$$

### Essay Question

*11) Our crotchety old pal "Flat Earth Fred" refuses to believe that exoplanets (planets around stars other than the Sun) exist, since astronomers haven't been able to take pictures of them. Write an explanation, in simple terms that a non-specialist like Fred can understand, of either the radial velocity method or the transit method for finding planets.*

I won't try to write a complete essay (much less two, one on the radial velocity method and one on the transit method). I'll just remind you that the discovery of planets was discussed on Tuesday, March 4, if you want to go over your notes. (Also, "Flat Earth Fred" will be more readily persuaded by an explanation that is grammatically correct, properly spelled, and well punctuated!)