

ASTRONOMY 294Z
The History of the Universe
Professor Barbara Ryden

Problem Set # 5
Due Tuesday, February 19
at class time

NAME (please print clearly): _____

SCORE (instructor use only): _____

1) [20 points] Einstein showed that mass (M) and energy (E) are interchangeable: $E = Mc^2$, where c is the speed of light. This implies, for instance, that 1 kilogram of matter is equivalent to an energy $E = (1 \text{ kg}) \times (3 \times 10^8 \text{ m/sec})^2 = 9 \times 10^{16} \text{ kg m}^2/\text{sec}^2$. An energy of $1 \text{ kg m}^2/\text{sec}^2$ is known as 1 **joule**, for short. The **joule** is not a unit of energy that is used much in everyday life. To give you a sense of scale, burning one gallon of gasoline releases 1.3×10^8 joules (130 million joules) of energy.

Okay, enough background. Here's the question: If you were capable of converting mass to energy with 100% efficiency, how much mass M would you need to produce an energy $E = 1.3 \times 10^8$ joules? [Hint: The number you get should be much, much less than the mass of a gallon of gasoline; burning gasoline is a sadly inefficient way of producing energy.]

2) [20 points] The total annual energy consumption in the United States of America is 10^{20} joules. If all this energy were produced by burning gasoline, how many gallons of gasoline would be required in one year? Hoover Reservoir, just northeast of Columbus, has a capacity of 2.1×10^{10} gallons. If Hoover Reservoir were filled to the brim with gasoline, would that be enough gasoline to supply the U.S.A.'s energy consumption for one year?

3) [20 points] The total annual energy consumption in the U.S.A. is 10^{20} joules, as mentioned in the previous problem. If you were capable of converting mass to energy with 100% efficiency, how much mass M would you need to produce an energy $E = 10^{20}$ joules? An adult male African elephant has a mass $M = 5000$ kilograms; if the elephant's mass were converted to energy with 100% efficiency, would that be enough to supply the U.S.A.'s energy consumption for one year?

[No actual elephants were harmed in the writing of this problem.]

4) [20 points] The Sun actually *does* convert mass into energy; it does this by nuclear fusion. During one second, the Sun produces an energy $E = 3.9 \times 10^{26}$ joules, which then is carried away by photons. How much mass M must the Sun convert into energy E each second?

5) [20 points] In the previous problem, you computed the mass M that the Sun converts into energy each second. The energy is then carried away by photons into the darkness of interstellar space. Thus, every second, the mass of the Sun is becoming smaller by an amount M . The current mass of the Sun is $M_{\text{sun}} = 2 \times 10^{30}$ kilograms. Using the value of M (the mass lost in one second) computed in problem #4, calculate how many seconds it will be before the Sun's mass drops to zero. Is this length of time greater than or less than the Hubble time, $1/H_0 = 4.4 \times 10^{17}$ sec?