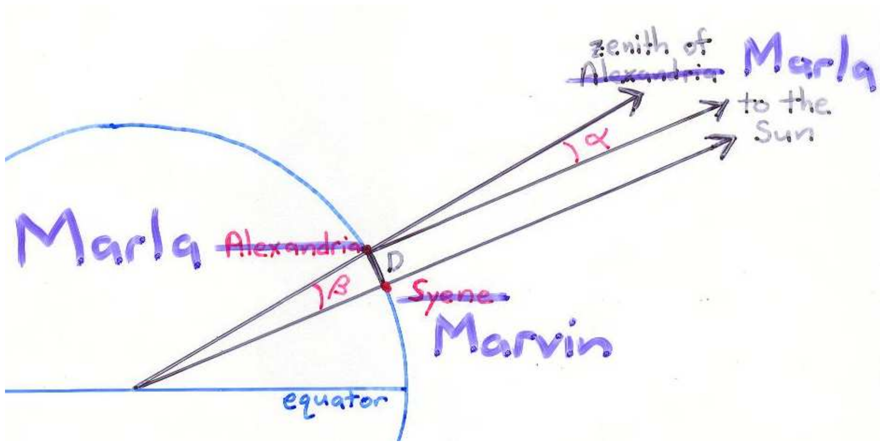
ASTRONOMY 294Z: The History of the Universe
Professor Barbara Ryden

SOLUTIONS TO PROBLEM SET # 1

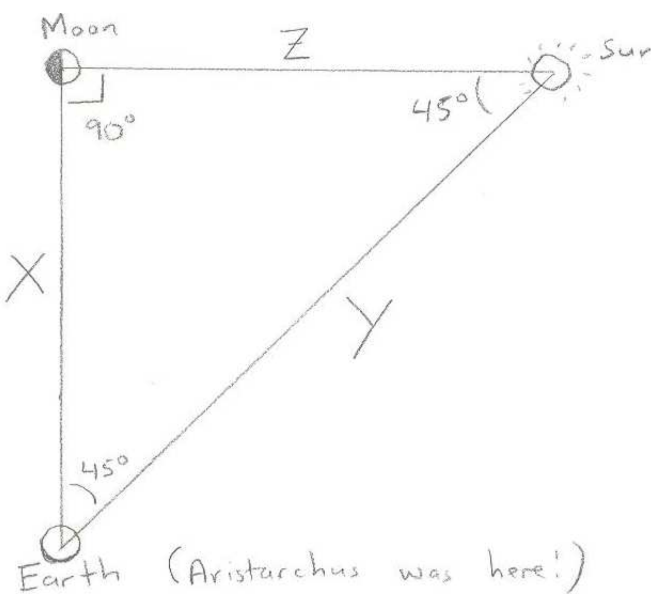
1) [20 points] *Two Martian astronomers, Marvin and Marla, are located due north and south of each other on the planet Mars. Marvin sees the Sun directly overhead (at the zenith) at noon. At the same time, Marla sees the Sun 6 degrees away from the zenith. Marla is 355 kilometers north of Marvin. Compute the circumference of the planet Mars.*



Since this is the method of Eratosthenes transplanted to Mars, I can reuse the "Eratosthenes" slide from the lecture of January 3, with a bit of relabeling (see above). The angle α is equal to $6°$. Since the angle α is equal to the angle β, the angle β is also equal to $6°$, or $1/60$ of a full circle. Thus, the distance $D = 355\,\text{km}$ is equal to $1/60$ of the circumference of Mars. The circumference C of the planet Mars is then

$$C = 60D = 60(355\,\text{km}) = 21{,}300\,\text{km}\ . \qquad (1)$$

2) [20 points] *Suppose that Aristarchus had measured an angle of 45 degrees between the Sun and the Moon when the Moon was in its first quarter phase. Draw an accurate diagram of the positions of Earth, Moon, and Sun necessary for this measurement to be correct. In this case, what is the ratio of the Earth – Sun distance to the Earth – Moon distance?*



My diagram of the Earth, Moon, and Sun is shown above. The Earth – Moon – Sun angle must be 90° for the Moon to be in its first quarter phase. The Moon – Earth – Sun angle is 45°, by supposition. The Moon – Sun – Earth angle must then be 45°, since the angles at the vertices of any triangle add to 180°. There are many ways of computing the ratio y/x, where y is the Earth – Sun distance and x is the Earth – Moon distance:

1) On my accurate scale drawing (above), I measure $y = 87\,\text{mm}$ and $x = 61\,\text{mm}$. Thus, $y/x = 87/61 = 1.43$.

2) By symmetry, $x = z$ in the above drawing. Since the square of the hypotenuse equals the sum of the square of the other two sides (remember the Pythagorean theorem!),

$$y^2 = x^2 + z^2 = x^2 + x^2 = 2x^2 \ , \qquad (2)$$

and thus $y = \sqrt{2}x$, leading to $y/x = \sqrt{2} = 1.414...$

3) Trigonometry mavens will note that $y/x = 1/\cos 45° = 1/0.707 = 1.414$.

3) [20 points] *If the Earth – Moon distance were* ***greater than*** *the Earth – Sun distance would an observer on the Earth be able to see the Moon in its first quarter phase? If your answer is "yes", draw a diagram showing how this could be true. If your answer is "no", explain why a first quarter Moon would be impossible.*

No. It would be impossible for us to see a first quarter Moon if the Earth – Moon distance (x in the diagram on the previous page) were greater than the Earth – Sun distance (y). When we see the Moon in its first quarter phase, the Earth – Moon – Sun angle must be a right angle; thus, the Earth – Sun distance y is the hypotenuse of a right triangle whenever Earthlings see a first quarter Moon. Since the hypotenuse is the longest side of the triangle, we must have $y > x$ when the Moon is in its first quarter phase.[1]

[1] Why is the hypotenuse the longest side? Remember the Pythagorean theorem: $y^2 = x^2 + z^2$. Since $z^2 > 0$, this means that $y^2 > x^2$, and thus $y > x$.

4) [20 points] *Modern measurements tell us that the average distance from the Earth to the Moon is 384,000 kilometers. Given that the Moon appears 1/2 degree across as seen from the Earth, what is the diameter of the Moon, in kilometers?*

In the lecture for January 8, I noted that a sphere that appears to be $1/2^\circ$ across must be at a distance x equal to 110 times its diameter d. Thus, given $x = 384{,}000\,\mathrm{km}$, we find

$$d = \frac{x}{110} = \frac{384{,}000\,\mathrm{km}}{110} = 3500\,\mathrm{km}\ . \tag{3}$$

(Parenthetical comment: if you are a trigonometry maven, you may have computed that the exact ratio of distance to diameter is $x/d = 114.6$, and computed the more accurate result $d = 3350\,\mathrm{km}$.)

5) [20 points] *Given that the circumference of the Earth is 40,000 kilometers, what is the Earth's diameter in kilometers? Given that there are 0.621 miles per kilometer, what is the Earth's diameter in miles? Given the results of problem (4), what is the ratio of the Earth's diameter to the Moon's diameter?*

The diameter d of a sphere is equal to its circumference divided by π. Thus, for the Earth,

$$d_{\rm earth} = \frac{40{,}000\,{\rm km}}{\pi} = 12,700\,{\rm km}\ . \tag{4}$$

Expressed in miles rather than kilometers, this becomes

$$d_{\rm earth} = 12{,}700\,{\rm km}\left(\frac{0.621\,{\rm miles}}{1\,{\rm km}}\right) = 7910\,{\rm miles}\ . \tag{5}$$

The ratio of the Earth's diameter to the Moon's diameter is

$$\frac{d_{\rm earth}}{d_{\rm moon}} = \frac{12{,}700\,{\rm km}}{3500\,{\rm km}} = 3.6\ . \tag{6}$$