ASTRONOMY 294Z: The History of the Universe
Professor Barbara Ryden

SOLUTIONS TO PROBLEM SET # 5

1) [20 points] *Einstein showed that mass (M) and energy (E) are interchangeable: $E = Mc^2$, where c is the speed of light. This implies, for instance, that 1 kilogram of matter is equivalent to an energy* $E = (1\,\mathrm{kg}) \times (3 \times 10^8\,\mathrm{m/sec})^2 = 9 \times 10^{16}\,\mathrm{kg\,m^2/sec^2}$. *An energy of* $1\,\mathrm{kg\,m^2/sec^2}$ *is known as 1 joule, for short. The joule is not a unit of energy that is used much in everyday life. To give you a sense of scale, burning one gallon of gasoline releases* $1.3 \times 10^8\,\mathrm{joules}$ *(130 million joules) of energy.*

Okay, enough background. Here's the question: If you were capable of converting mass to energy with 100% efficiency, how much mass M would you need to produce an energy $E = 1.3 \times 10^8\,\mathrm{joules}$*?*

If $E = Mc^2$, then $M = E/c^2$. Thus, to produce $E = 1.3 \times 10^8\,\mathrm{joules} = 1.3 \times 10^8\,\mathrm{kg\,m^2/sec^2}$, the amount of mass required is

$$M = \frac{E}{c^2} = \frac{1.3 \times 10^8\,\mathrm{kg\,m^2/sec^2}}{(3 \times 10^8\mathrm{m/sec})^2} = 1.44 \times 10^{-9}\,\mathrm{kg}\ . \qquad (1)$$

This mass can also be written as 1.44 micrograms; it's roughly equivalent to the mass of a single grain of sand 0.1 millimeter across.

2) [20 points] *The total annual energy consumption in the U.S.A. is* 10^{20} joules. *If all this energy were produced by burning gasoline, how many gallons of gasoline would be required in one year? Hoover Reservoir, just northeast of Columbus, has a capacity of* 2.1×10^{10} *gallons. If Hoover Reservoir were filled to the brim with gasoline, would that be enough gasoline to supply the U.S.A.'s energy consumption for one year?*

The energy released by burning gasoline is 1.3×10^{8} joules/gallon, as noted in the first problem. Thus, the total volume of gasoline that must be burned to release 10^{20} joules is

$$V = \frac{10^{20}\,\text{joules}}{1.3 \times 10^{8}\,\text{joules/gallon}} = 7.7 \times 10^{11}\,\text{gallons}\ . \qquad (2)$$

This is nearly 37 times the capacity of Hoover Reservoir. Thus, even if the reservoir were filled to the brim with gasoline, it would *not* be enough to supply one year's energy consumption. In fact, it would be drained dry in 10 days!

3) [20 points] *The total annual energy consumption in the U.S.A. is* 10^{20} joules*, as mentioned in the previous problem. If you were capable of converting mass to energy with 100% efficiency, how much mass M would you need to produce an energy* $E = 10^{20}$ joules*? An adult male African elephant has a mass* $M = 5000$ kilograms*; if the elephant's mass were converted to energy with 100% efficiency, would that be enough to supply the U.S.A.'s energy consumption for one year?*

If conversion from mass to energy were totally efficient, the amount of mass required to create $E = 10^{20}\,\text{joules} = 10^{20}\,\text{kg}\,\text{m}^2/\text{sec}^2$ would be

$$M = \frac{E}{c^2} = \frac{10^{20}\,\text{kg}\,\text{m}^2/\text{sec}^2}{(3 \times 10^8\,\text{m}/\text{sec})^2} = 1100\,\text{kg}\ , \tag{3}$$

or slightly more than one metric ton. Thus is less than 1/4 the mass of an adult male African elephant, so one elephant-mass *would* be enough to supply one year's energy consumption.

4) [20 points] *The Sun actually* does *convert mass into energy; it does this by nuclear fusion. During one second, the Sun produces an energy* $E = 3.9 \times 10^{26}$ joules, *which then is carried away by photons. How much mass* M *must the Sun convert into energy* E *each second?*

The amount of mass required to create $E = 3.9 \times 10^{26}$ joules $= 3.9 \times 10^{26}\,\mathrm{kg\,m^2/sec^2}$ is

$$M = \frac{E}{c^2} = \frac{3.9 \times 10^{26}\,\mathrm{kg\,m^2/sec^2}}{(3 \times 10^8\,\mathrm{m/sec})^2} = 4.33 \times 10^9\mathrm{kg}\ . \tag{4}$$

In other units, this is nearly 900,000 elephant-masses.

5) [20 points] *In the previous problem, you computed the mass M that the Sun converts into energy each second. The energy is then carried away by photons into the darkness of interstellar space. Thus, every second, the mass of the Sun is becoming smaller by an amount M. The current mass of the Sun is* $M_{\rm sun} = 2 \times 10^{30}$ kilograms. *Using the value of M (the mass lost in one second) computed in problem #4, calculate how many seconds it will be before the Sun's mass drops to zero. Is this length of time greater than or less than the Hubble time,* $1/H_0 = 4.4 \times 10^{17}$ sec*?*

The Sun is losing mass at the rate 4.33×10^9 kg/sec, as we computed in the previous problem. Although losing 900,000 elephants per second seems like a large mass rate, the Sun's current mass is huge: $M_{\rm sun} = 2 \times 10^{30}$ kg. Thus, the time it would take the Sun to dwindle to nothing at its current mass loss rate is

$$t = \frac{2 \times 10^{30}\,{\rm kg}}{4.33 \times 10^9\,{\rm kg/sec}} = 4.6 \times 10^{20}\,{\rm sec} = 14\,{\rm trillion\ years}\ . \tag{5}$$

This is over a thousand times *greater than* the Hubble time.