

ASTRONOMY 294Z: The History of the Universe
Professor Barbara Ryden

SOLUTIONS TO PROBLEM SET # 6

1) [20 points] *Potassium-40 is an unstable atomic nucleus; it decays to argon-40 with a half-life of 1.3 billion years. Suppose that a rock contains 1,000,000 potassium-40 atoms at the time it forms. How many potassium-40 atoms will be left after 1.3 billion years? How many will be left after 2.6 billion years? How many will be left after 3.9 billion years?*

After one half-life (1.3 billion years), there will be 500,000 potassium-40 atoms left.

After two half-lives (2.6 billion years), there will be 250,000 potassium-40 atoms left.

After three half-lives (3.9 billion years), there will be 125,000 potassium-40 atoms left.

2) [20 points] The “lifespan” of the Sun is 10 billion years; that is, at the time it formed, it contained enough hydrogen to power nuclear fusion for 10 billion years. The star Altair, like the Sun, is powered by the fusion of hydrogen to helium. The mass of Altair is $M_{\text{altair}} = 1.7M_{\text{sun}}$. The luminosity of Altair is $L_{\text{altair}} = 10.7L_{\text{sun}}$. Is the lifespan of Altair shorter or longer than that of the Sun? What is the approximate lifespan of Altair, in billions of years?

The lifespan of a star is directly proportional to its mass; the mass of a star represents its fuel supply, so doubling the mass, all other things being equal, will double its lifespan. In addition, the lifespan of a star is *inversely* proportional to its luminosity; the luminosity of a star tells you how fast it is using its fuel supply, so doubling the luminosity, all other things being equal, will halve its lifespan. If we scale everything by the Sun’s properties, we can write a star’s lifespan t_{star} as

$$t_{\text{star}} = t_{\text{sun}} \left(\frac{M_{\text{star}}}{M_{\text{sun}}} \right) \left(\frac{L_{\text{sun}}}{L_{\text{star}}} \right) , \quad (1)$$

with $t_{\text{sun}} = 10$ billion years, as given in the problem. Since Altair’s luminosity is more than 10 times the Sun’s luminosity, while its mass is only 70% greater than the Sun’s mass, the lifespan of Altair will be *shorter* than that of the Sun. Thus, Altair’s lifespan will be

$$t_{\text{altair}} = 10 \text{ billion years} (1.7) \left(\frac{1}{10.7} \right) = 1.6 \text{ billion years} . \quad (2)$$

3) [20 points] *Exponential growth (as we saw while discussing inflation during the early universe) can be very effective at making small things into big things. Suppose that you invest \$1000 in an investment that doubles every 10 years. That is, after 10 years, your investment will be worth \$2000, after 20 years, it will be worth \$4000, and so forth. What will be the value of your investment after 50 years? What will be its value of your investment after 100 years? What will be the value of your investment after 200 years? What will be the value of your investment after 300 years?*

I'll start by tabulating the value of the investment in kilobucks (1 kilobuck = \$1000) as a function of time in decades:

Time	Value
0 decades	1 kilobuck
1 decade	2 kilobucks
2 decades	4 kilobucks
3 decades	8 kilobucks
4 decades	16 kilobucks
5 decades	32 kilobucks

Thus, after 50 years, the value of the investment is \$32,000. I could simply continue the table, but I can save paper by noting that x , the value of the investment in kilobucks, is related to the elapsed time t , measured in decades, by the relation $x = 2^t$. (Compare to the above table for verification.) Thus, after 100 years (10 decades), the value of the investment is $x = 2^{10}$ kilobucks = \$1,024,000. After 200 years (20 decades), the value of the investment is $x = 2^{20}$ kilobucks = \$1,048,576,000. After 300 years (30 decades), the value of the investment is $x = 2^{30}$ kilobucks = \$1,073,741,824,000. If only we had time machines, we could become trillionaires by traveling back in time to the reign of Queen Anne, and investing the 17th century equivalent of one thousand dollars. (By the way, doubling your money every decade is equivalent to an investment that pays 7% interest, compounded monthly, so we're not talking outrageously high interest rates here.)

4) [40 points] *The star Phi Orionis, like the Sun, is powered by the fusion of hydrogen to helium. The mass of Phi Orionis is $M_{\text{phi}} = 18M_{\text{sun}}$. The luminosity of Phi Orionis is $L_{\text{phi}} = 20,000L_{\text{sun}}$. Discuss the likelihood of intelligent life existing on a planet orbiting the star Phi Orionis. [Questions you might want to consider: What is the lifespan of Phi Orionis? How long did it take intelligent life to develop on Earth? How far would you have to be from Phi Orionis to receive the same flux of light that we receive here on Earth from the Sun?]*

It is unlikely, in my opinion, that intelligent life has developed from scratch on a planet orbiting Phi Orionis. First, given the mass and luminosity of Phi Orionis, its expected lifespan is (see problem 2)

$$t_{\text{phi}} = t_{\text{sun}} \left(\frac{M_{\text{phi}}}{M_{\text{sun}}} \right) \left(\frac{L_{\text{sun}}}{L_{\text{phi}}} \right) = 10 \text{ billion years} (18) \left(\frac{1}{20,000} \right) = 9 \text{ million years} . \quad (3)$$

Since it has taken over 4.5 billion years for (more-or-less) intelligent life to develop on Earth, having it evolve in a time of less than 9 million years implies an extraordinarily rapid rate of evolution.

Second, since the luminosity of Phi Orionis is 20,000 times that of the Earth, planets close to Phi Orionis would be badly scorched. For a planet to receive the same flux of light from Phi Orionis that we receive 1 AU from the Sun, its distance from Phi Orionis would have to be $\sqrt{20,000} \text{ AU} = 141 \text{ AU}$. This is far larger than the orbit of Neptune, in a region where the Solar System has no planets at all.