ASTRONOMY 294Z: The History of the Universe
Professor Barbara Ryden

SOLUTIONS TO PROBLEM SET # 7

1) [20 points] *The age of the universe (that is, the time since the Big Bang) is 14 billion years. The age of the Solar System is 4.56 billion years. Thus, the Solar System has existed for 32.6% of the age of the universe. For what percentage of the total age of the universe have the following things existed?*

a) Helium nuclei have been around since the time of primordial nucleosynthesis, at a time $t_{\rm pn} \approx 7\,{\rm min} \approx 420\,{\rm sec}$ after the Big Bang. The age of the universe, expressed in seconds, is

$$t_{\rm uni} = 1.4 \times 10^{10}\,{\rm years} \left(\frac{3.16 \times 10^7\,{\rm sec}}{1\,{\rm year}} \right) = 4.4 \times 10^{17}\,{\rm sec}\ . \qquad (1)$$

The fraction of the age of the universe during which helium nuclei have been around is

$$F = \frac{t_{\rm uni} - t_{\rm pn}}{t_{\rm uni}} = 1 - \frac{t_{\rm pn}}{t_{\rm uni}} = 1 - \frac{420\,{\rm sec}}{4.4 \times 10^{17}\,{\rm sec}} = 1 - 10^{-15} = 0.999999999999999\ . \qquad (2)$$

Expressed as a percentage, this is **99.9999999999999%** of the age of the universe. (There should be 13 nines after the decimal point – I hope I counted them correctly!)

b) Neutral atoms have been around since the universe became transparent, at a time $t_{\rm trans} = 350{,}000\,{\rm years}$ after the Big Bang. The fraction of the age of the universe during which neutral atoms have been around is

$$F = 1 - \frac{t_{\rm trans}}{t_{\rm uni}} = 1 - \frac{3.5 \times 10^5\,{\rm years}}{1.4 \times 10^{10}\,{\rm years}} = 1 - 0.000025 = 0.999975\ . \qquad (3)$$

Expressed as a percentage, this is **99.9975%** of the age of the universe.

c) Galaxies have been around since the universe had an age $t_{\rm gal} = 5 \times 10^8$ years. The fraction of the age of the universe during which galaxies have been present is

$$F = 1 - \frac{t_{\rm gal}}{t_{\rm uni}} = 1 - \frac{5 \times 10^8\,{\rm years}}{1.4 \times 10^{10}\,{\rm years}} = 1 - 0.0357 = 0.964\ . \qquad (4)$$

Expressed as a percentage, this is **96.4%** of the age of the universe.

d) Dating from the Declaration of Independence, the U.S.A. has existed for 231.7 years. Written as a fraction of the age of the universe, the age of the U.S.A. is

$$F = \frac{231.7\,\text{years}}{1.4 \times 10^{10}\,\text{years}} = 1.66 \times 10^{-8} \ . \tag{5}$$

Expressed as a percentage, this is **0.00000166%** of the age of the universe.

e) Dating from my birth, I have existed for 46.8 years. Written as a fraction of the age of the universe, my age is

$$F = \frac{46.8\,\text{years}}{1.4 \times 10^{10}\,\text{years}} = 3.3 \times 10^{-9} \ . \tag{6}$$

Expressed as a percentage, this is **0.00000033%** of the age of the universe. (Gosh, that makes me feel young!)

2) [20 points] *The Whirlpool Galaxy is at a distance* $d = 7.1\,\mathrm{Mpc}$ *from us. Using Hubble's law, what do you expect the radial velocity* v *of the Whirlpool Galaxy to be? What do you expect the redshift* z *of the Whirlpool Galaxy to be? When hydrogen is at rest, it produces an emission line with a wavelength* $\lambda_0 = 656.281\,\mathrm{nanometers}$*; what wavelength* λ *would you measure for the corresponding emission line from hydrogen in the Whirlpool Galaxy?*

The radial velocity v is given by Hubble's Law:

$$v = H_0 d = (70\,\mathrm{km/sec/Mpc})(7.1\,\mathrm{Mpc}) = 497\,\mathrm{km/sec}\ . \tag{7}$$

The redshift $z = (\lambda - \lambda_0)/\lambda_0$ is equal to the radial velocity divided by the speed of light:

$$z = \frac{v}{c} = \frac{497\,\mathrm{km/sec}}{300{,}000\,\mathrm{km/sec}} = 0.00166\ . \tag{8}$$

The wavelength λ is given by the relation

$$z = \frac{\lambda - \lambda_0}{\lambda_0}\ , \tag{9}$$

which can be rearranged to give

$$z\lambda_0 + \lambda_0 = \lambda\ . \tag{10}$$

Thus, the observed wavelength λ will be

$$\lambda = (z+1)\lambda_0 = (1.00166)(656.281\,\mathrm{nm}) = 657.368\,\mathrm{nm}\ . \tag{11}$$

3) [20 points] *We can detect a star with our naked eyes as long as its flux is above some minimum threshold,* $F_{\rm min}$. *The flux of the Sun would be equal to* $F_{\rm min}$ *if it were at a distance of 17 parsecs from us. In other words, the maximum distance at which you would be able to see the Sun with your naked eyes is* $d_{\rm sun} = 17\,{\rm pc}$. *The luminosity of a supernova (that is, an exploding star) is* $L_{\rm super} = 3.6 \times 10^9 L_{\rm sun}$. *What is the maximum distance* $d_{\rm super}$ *at which you would be able to see a supernova with your naked eyes? If a supernova went off in the Andromeda Galaxy, would we be able to see it here on Earth without the aid of a telescope?*

Since the Sun has the barely detectable flux $F_{\rm min}$ at a distance $d_{\rm sun} = 17\,{\rm pc}$, we may write

$$F_{\rm min} = \frac{L_{\rm sun}}{4\pi d_{\rm sun}^2} \; . \tag{12}$$

A supernova will have the same flux $F_{\rm min}$ at a larger distance, $d_{\rm super}$. The value of $d_{\rm super}$ is dictated by the relation

$$F_{\rm min} = \frac{L_{\rm super}}{4\pi d_{\rm super}^2} \; . \tag{13}$$

By equating the right-hand sides of equations (12) and (13), we find that

$$\frac{L_{\rm sun}}{4\pi d_{\rm sun}^2} = \frac{L_{\rm super}}{4\pi d_{\rm super}^2} \; . \tag{14}$$

After canceling the factors of 4π, we may rewrite this equation as

$$d_{\rm super}^2 = \frac{L_{\rm super}}{L_{\rm sun}} d_{\rm sun}^2 \; . \tag{15}$$

Taking the square root of both sides of the equation, we find

$$d_{\rm super} = \sqrt{\frac{L_{\rm super}}{L_{\rm sun}}} d_{\rm sun} = \sqrt{3.6 \times 10^9}(17\,{\rm pc}) = 1{,}020{,}000\,{\rm pc} \; . \tag{16}$$

A supernova can be seen by your unaided eye when it's a million parsecs away! Since the Andromeda Galaxy is 700,000 parsecs away (as you may recall from the midterm), a supernova exploding in the Andromeda Galaxy *can* be seen with your naked eyes.

4) [40 points] *Our old friend "Flat-Earth Fred" is up to some new tricks. He now believes that the Big Bang Model is bogus; he thinks that he lives in a static universe that is both infinitely large and eternally old. Describe what evidence you could provide that would convince Fred that the universe* cannot *be static, infinitely large, and eternally old. (Remember, skeptical Fred prefers evidence that he can see directly with his own eyes.)*

I won't try to write a complete and polished essay here; I'll just point out that Fred, given his preference for evidence he can see with his own eyes, will probably be most strongly convinced by looking up at the night sky while you provide him with a concise, yet clear, explanation of Olbers' paradox.