

# Chapter 11

## The Solar Wind

A stellar wind consists of particles emitted from the stellar atmosphere with a sufficiently large velocity to escape the star's gravitational attraction. The escape velocity at the surface of a star with mass  $M_*$  and radius  $R_*$  is

$$v_{\text{esc}} = \left( \frac{2GM_*}{R_*} \right)^{1/2} = 620 \text{ km s}^{-1} (M_*/M_\odot)^{1/2} (R_*/R_\odot)^{-1/2}. \quad (11.1)$$

Since a star will tend to accrete mass, due to its gravitational attraction, some non-gravitational force is needed to counteract the inward pull of gravity, and accelerate the outermost layers of the stellar atmosphere away from the star. This wind-driving force can be, depending on the type of wind, a gradient in the gas pressure, a gradient in the radiation pressure, or a gradient in the magnetic pressure.

Observed stellar winds may be placed into one of four main categories.

1. *Winds from main sequence stars.* Stars similar to the sun have a low rate of mass loss:  $\dot{M} \sim 10^{-14} M_\odot \text{ yr}^{-1}$ . ( $1 M_\odot \text{ yr}^{-1} = 6.3 \times 10^{25} \text{ g sec}^{-1}$ .) The asymptotic velocity  $u_\infty$  of the wind is comparable to the escape velocity  $v_{\text{esc}}$ . Winds from solar-type stars are thought to be driven by gas pressure gradients in the corona. The prototypical star in this category is the sun.
2. *Winds from hot, luminous stars.* Stars with  $T_{\text{eff}} \gtrsim 15,000 \text{ K}$  and  $L \gtrsim 3000 L_\odot$  have a high rate of mass loss:  $\dot{M} \sim 10^{-7} \rightarrow 10^{-5} M_\odot \text{ yr}^{-1}$ . Their winds are very fast, with  $u_\infty > v_{\text{esc}}$ . Winds from hot stars are thought to be driven by radiation pressure. Wolf-Rayet stars fall

into this category. WR stars are hot, luminous stars with extended envelopes; their mass loss rates ( $\dot{M} \sim 10^{-5} M_{\odot} \text{ yr}^{-1}$ ) and wind terminal velocities ( $u_{\infty} \sim 2500 \text{ km s}^{-1}$ ) are among the largest observed for any type of star.

3. *Winds from cool, luminous stars.* Stars with  $T_{\text{eff}} \lesssim 6000 \text{ K}$  and  $L \gtrsim 100 L_{\odot}$  have a high rate of mass loss:  $\dot{M} \sim 10^{-8} \rightarrow 10^{-5} M_{\odot} \text{ yr}^{-1}$ . The wind velocity, however, is very low, with  $u_{\infty} < v_{\text{esc}}$ . The mechanism that drives the wind from cool stars is uncertain; a leading candidate is radiation pressure on dust grains, aided by stellar pulsations in the outer atmosphere. Stars in this category are K and M giants and supergiants.
4. *Winds from extremely young stars.* T Tauri stars have mass loss rates of  $\dot{M} \sim 10^{-9} \rightarrow 10^{-7} M_{\odot} \text{ yr}^{-1}$ . The wind velocity is  $u_{\infty} \sim 200 \text{ km s}^{-1}$ . T Tauri stars have circumstellar disks that might be accretion disks. The strength of the wind is correlated with the luminosity of the disk; this suggests that the outgoing wind might be powered by the accretion disk.

Figure 11.1 shows the location on a Hertzsprung-Russell diagram of stars with high rates of mass loss. Hot, luminous stars with winds (including Wolf-Rayet stars) are in the upper left; cool, luminous stars with winds are in the upper right. The approximate location of T Tauri stars on the H-R diagram is also shown.

The best-studied stellar wind is the solar wind, which is the weakest of all measured stellar winds. The solar wind consists mainly of ionized hydrogen and fully ionized helium, with heavier elements present in solar abundances. At 1 AU from the sun, the solar wind is supersonic ( $u > a$ ) and super-Alfvénic ( $u > v_A$ ). As shown in Figure 11.2, the solar wind is “gusty”, with significant variations in velocity and density. As a lowest-order approximation, we can recognize two distinct types of solar wind: a high-speed wind and a low-speed wind. In the high-speed wind, the mean proton velocity is  $u_p = 700 \text{ km s}^{-1}$  and the mean proton density is  $n_p = 3.4 \text{ cm}^{-3}$ . The gas pressure is  $P = 1.9 \times 10^{-10} \text{ dyne cm}^{-2}$  and the magnetic pressure is  $B^2/(8\pi) = 1.7 \times 10^{-10} \text{ dyne cm}^{-2}$ ; both of these pressures are much smaller than the kinetic energy density  $\rho u^2/2 = 1.4 \times 10^{-8} \text{ erg cm}^{-3}$ . In the low-speed wind, the mean proton velocity is  $u_p = 330 \text{ km s}^{-1}$  and the mean proton density is  $n_p = 10.3 \text{ cm}^{-3}$ . The gas pressure is  $P = 2.6 \times 10^{-10} \text{ dyne cm}^{-2}$

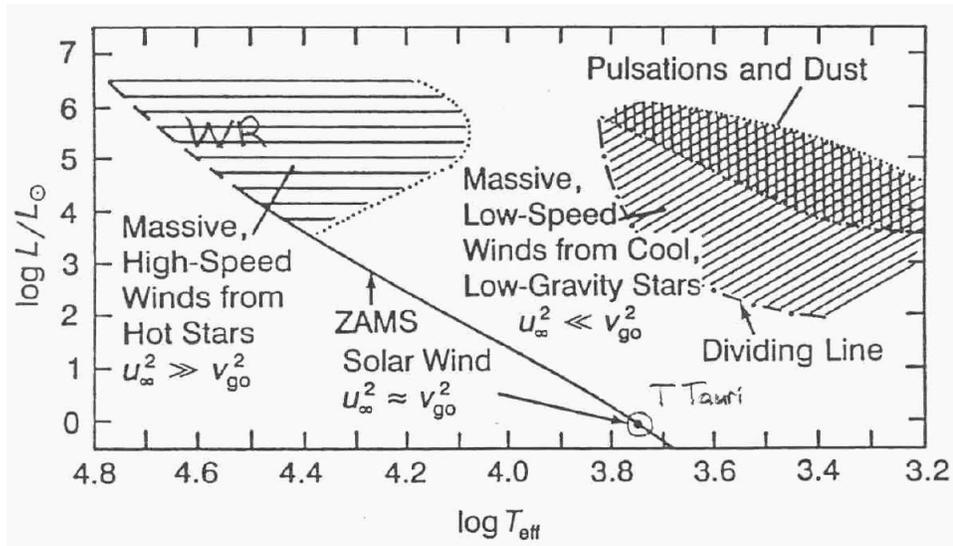


Figure 11.1: A Hertzsprung-Russell diagram (luminosity versus effective temperature) showing the location of stars with high rates of mass loss from stellar winds.

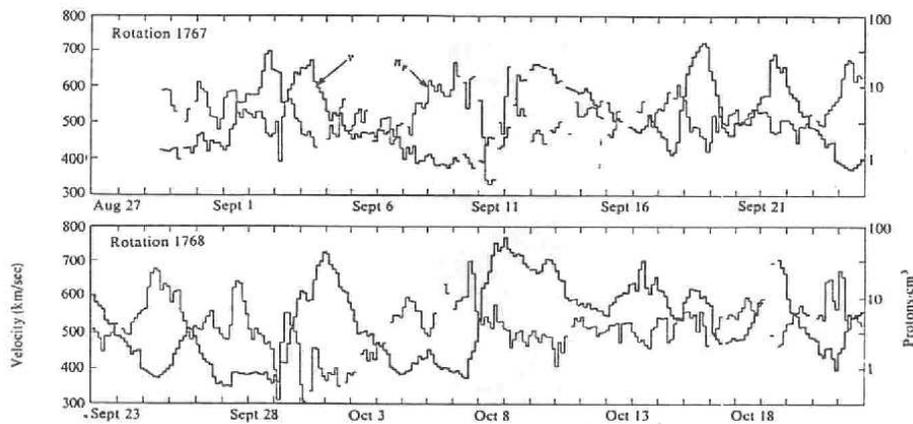


Figure 11.2: The velocity and density of the solar wind at  $R = 1$  AU from the Sun, as measured by the Mariner 2 spacecraft.

and the magnetic pressure is  $B^2/(8\pi) = 1.7 \times 10^{-10}$ ; much smaller than the kinetic energy density  $\rho u^2/2 = 9.4 \times 10^{-9} \text{ erg cm}^{-3}$ .

At any given time, part of the sun's corona is emitting a low-speed wind, and part is emitting a high-speed wind. (The high-speed winds appear to come from "coronal holes" – regions of low density and low temperature where the magnetic field lines are not closed.) Both the high and low speed winds produce a proton flux of  $\sim 3 \times 10^8 \text{ protons cm}^{-2} \text{ sec}^{-1}$  at 1 AU. This leads to a total mass loss rate of  $2 \times 10^{-14} M_{\odot} \text{ yr}^{-1}$ .

The study of stellar atmospheres usually starts with the statement, "We assume that the atmosphere is in hydrostatic equilibrium." Well, that statement is fraudulent – a stellar atmosphere *cannot* be in hydrostatic equilibrium. There is always going to be a loss of high velocity particles. To show that the sun's corona is not in hydrostatic equilibrium, we first assume that it is in equilibrium, and then show that the assumption leads to an absurd conclusion. The corona is the outermost part of the sun's atmosphere; it extends from a distance of 2000 km above the sun's visible surface out into interplanetary space. At the base of the corona,  $r_0 = 7 \times 10^{10} \text{ cm}$ , the temperature is  $T_0 = 2 \times 10^6 \text{ K}$  and the number density of protons is  $n_0 = 10^8 \text{ cm}^{-3}$ . The resulting gas pressure is  $P_0 = 2n_0kT_0 = 0.06 \text{ dyne cm}^{-2}$ . At the high temperatures present in the corona, the thermal structure is determined by heat conduction. The coefficient of thermal conductivity for an ionized gas is equal to

$$K(T) = 3 \times 10^9 \text{ g cm sec}^{-3} \text{ K}^{-1} \left( \frac{T}{2 \times 10^6 \text{ K}} \right)^{5/2}. \quad (11.2)$$

In the corona, where there are no local heat sources, the fact that heat flow is in a steady state tells us that

$$\vec{\nabla} \cdot (K \vec{\nabla} T) = \frac{1}{r^2} \frac{d}{dr} \left( r^2 K \frac{dT}{dr} \right) = 0. \quad (11.3)$$

In conjunction with the relation  $K \propto T^{5/2}$ , this yields

$$T(r) = T_0 \left( \frac{r}{r_0} \right)^{-2/7}. \quad (11.4)$$

In general, if  $K \propto T^n$ ,  $T \propto r^{-1/(n+1)}$ ; so if the corona were neutral, with  $n = 1/2$ , the temperature would fall off at the more rapid rate  $T \propto r^{-2/3}$ .

If, as we are assuming, the corona is in hydrostatic equilibrium, we have the familiar equation

$$\frac{1}{\rho} \frac{dP}{dr} = -\frac{GM}{r^2} . \quad (11.5)$$

If the corona consists of ionized hydrogen,  $\rho \approx m_p n$  and  $P = 2nkT$ . Using the substitution  $T = T_0(r/r_0)^{-2/7}$ ,

$$\frac{d}{dr} \left[ n \left( \frac{r}{r_0} \right)^{-2/7} \right] = -\frac{GMm_p}{2kT_0} \frac{n}{r^2} . \quad (11.6)$$

This equation has the solution

$$n(r) = n_0 x^{2/7} \exp \left[ \frac{7}{5} \frac{r_0}{h} (x^{-5/7} - 1) \right] , \quad (11.7)$$

where  $x \equiv r/r_0$  and the scale height  $h$  is given by the relation

$$h \equiv \frac{2kT_0 r_0^2}{GMm_p} = 1.2 \times 10^{10} \text{ cm} \left( \frac{T_0}{2 \times 10^6} \right) \left( \frac{r_0}{R_\odot} \right)^2 \left( \frac{M}{M_\odot} \right)^{-1} . \quad (11.8)$$

Near the base of the corona, where  $r - r_0 \ll r_0$ , the density falls off exponentially, with

$$n(r) \approx n_0 e^{-(r-r_0)/h} . \quad (11.9)$$

At large radii, the density increases at the rate  $n \propto r^{2/7}$ . In fact, at a distance of 1 AU, where  $r = 210r_0$ , the hydrostatic corona must have a number density  $n = 1.4 \times 10^{-3} n_0 \sim 10^5 \text{ cm}^{-3}$ . But this, of course, is much larger than the number density of protons that are actually observed at the earth's orbit.

The gas pressure as a function of radius is

$$P(r) = P_0 \exp \left[ \frac{7}{5} \frac{r_0}{h} (x^{-5/7} - 1) \right] . \quad (11.10)$$

To maintain the corona in hydrostatic equilibrium, there must be a finite pressure at large radii, with the value

$$P(\infty) = P_0 \exp \left[ -\frac{7}{5} \frac{r_0}{h} \right] \approx 2 \times 10^{-5} \text{ dyne cm}^{-2} . \quad (11.11)$$

The pressure required to keep the solar corona in hydrostatic equilibrium is much greater than the pressure  $P \approx 3 \times 10^{-13} \text{ dyne cm}^{-2}$  that is found in the interstellar medium.

Since the corona can't be stationary, let's approximate it as having a steady-state outflow. The continuity equation tells us that  $\dot{M} = 4\pi r^2 \rho u$ . This is similar to the continuity equation for spherical accretion; the only difference is in the sign convention. For accretion, we adopted  $\dot{M} > 0$  and  $u < 0$ ; for winds, we adopt  $\dot{M} > 0$  and  $u > 0$ . The equation for momentum conservation in an unmagnetized, spherical, steady-state wind is

$$u \frac{du}{dr} + \frac{a^2}{\rho} \frac{d\rho}{dr} + \frac{GM}{r^2} = 0. \quad (11.12)$$

The self-gravity of the wind has been ignored. Using the continuity equation to eliminate  $\rho$ , we find, once again, the Bondi equation:

$$\frac{1}{2} \left( 1 - \frac{a^2}{u^2} \right) \frac{d}{dr} (u^2) = -\frac{GM}{r^2} \left( 1 - \frac{2a^2 r}{GM} \right). \quad (11.13)$$

Care must be taken in choosing, from among the possible solutions of the Bondi equation, the solution that matches the boundary conditions of the flow. For accretion, we wanted solutions with  $u^2 \rightarrow 0$  as  $r \rightarrow \infty$ . For winds, we want solutions with  $u^2 \rightarrow 0$  as  $r \rightarrow 0$ . In the nomenclature of Chapter 8, a stellar wind must have a subsonic solution of type 3, or a transonic solution of type 2. The subsonic solutions, in this context, are known as “stellar breezes”; the gas in these flows never becomes supersonic or escapes from the gravitational well of the star. The transonic solution is known as the “Parker wind”. Since the solar wind is known to be supersonic at the earth's orbit, the “Parker wind” is the appropriate solution for the solar wind.

The temperature is nearly constant throughout the inner regions of the corona. This fact prompted Parker to look at *isothermal* winds. An isothermal wind of ionized hydrogen has the constant sound speed

$$a_0 = \left( \frac{2kT_0}{m_p} \right)^{1/2} = 180 \text{ km s}^{-1} \left( \frac{T_0}{2 \times 10^6 \text{ K}} \right)^{1/2}. \quad (11.14)$$

In the transonic solution, the wind velocity will be  $u = a_0$  at the sonic radius

$$r_s = \frac{GM}{2a_0^2} = 2.0 \times 10^{11} \text{ cm} \left( \frac{M}{M_\odot} \right) \left( \frac{T_0}{2 \times 10^6} \right)^{-1}. \quad (11.15)$$

The Bondi equation for an isothermal wind is

$$\left( 1 - \frac{a_0^2}{u^2} \right) \frac{d}{dr} \left( \frac{u^2}{a_0^2} \right) = -4 \frac{r_s}{r^2} \left( 1 - \frac{r}{r_s} \right). \quad (11.16)$$

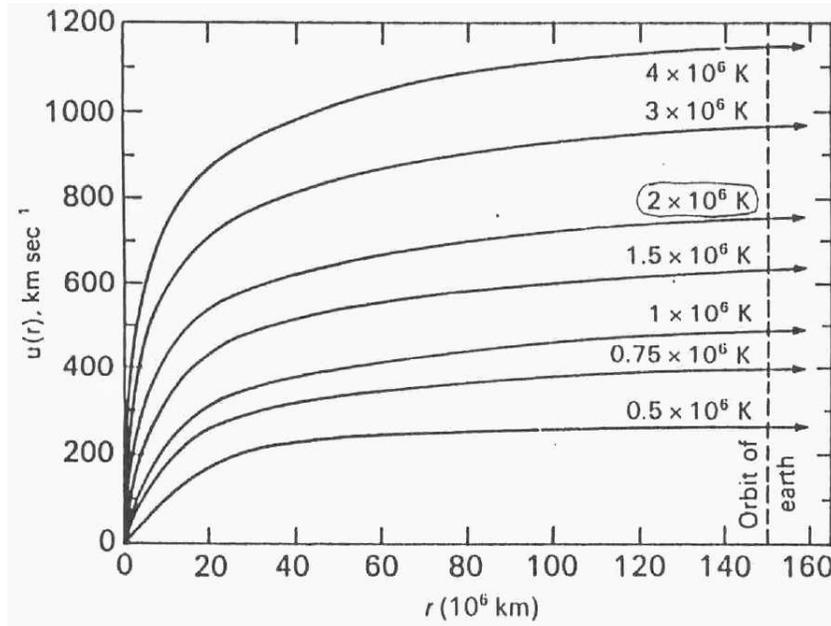


Figure 11.3: The wind velocity of an isothermal Parker wind, for different values of the temperature  $T_0$ .

This equation may be integrated, using the boundary restriction  $u = a_0$  at  $r = r_s$ , to yield the Bernoulli integral for an isothermal wind:

$$\frac{u^2}{a_0^2} - \ln \left( \frac{u^2}{a_0^2} \right) = 4 \ln \frac{r}{r_s} + 4 \frac{r_s}{r} - 3. \quad (11.17)$$

This equation gives the wind velocity  $u$  as a function of  $r$ ; a plot of  $u(r)$  for different temperatures is shown in Figure 11.3. At large radii, the wind velocity is approximately  $u \approx 2a_0[\ln(r/r_s)]^{1/2}$ . At small radii, the velocity is  $u \approx a_0 e^{3/2} \exp(-2r_s/r)$ . For the sun, the sonic radius  $r_s = 2 \times 10^{11}$  cm is only three times the radius of the base of the corona. At 1 AU, the radius is  $r = 75r_s$ , and the solar wind velocity predicted by the isothermal Parker model is  $u \approx 4.1a_0 \approx 740 \text{ km s}^{-1} (T_0/2 \times 10^6)^{1/2}$ . This velocity is approximately equal to the velocity actually measured for the solar wind (at least its high-speed component). The agreement, however, is partly due to chance. The solar wind is *not*, in fact, perfectly isothermal. At 1 AU, the temperature has dropped to  $1.5 \times 10^5$  K.

The magnetic field associated with the solar wind at the earth's orbit is

$B \approx 5 \times 10^{-5}$  G. The fact that  $\vec{\nabla} \cdot \vec{B} = 0$  implies that

$$B_r = B_0(r/r_0)^{-2} , \quad (11.18)$$

where  $B_0 = B_r(r_0)$ . Since the base of the solar corona is at a radius  $r_0 = 4.6 \times 10^{-3}$  AU, the radial component of the magnetic field at the base of the corona must be of magnitude  $B_0 \sim 1$  G. Since the corona and solar wind are fully ionized, the magnetic field lines are pinned to the solar wind as it streams outward. The ends of the magnetic field lines are anchored to the sun, as it rotates with an angular velocity  $\Omega_* = 3 \times 10^{-6} \text{ sec}^{-1}$ . If the magnetic pressures are negligibly small, and the ionized gas streams radially outward, then the magnetic field lines in the equatorial plane will be drawn outward in spirals. If the wind speed is constant, the magnetic field lines are Archimedean spirals, with the shape

$$r(\phi) - r_0 = \frac{u_r}{\Omega_*}(\phi - \phi_0) . \quad (11.19)$$

The above analysis assumes that the magnetic force terms in the equations of motion are negligibly small, and that the magnetic field lines are therefore passively carried about by the gas flow. The magnetic force terms in the radial direction are, in fact, very small compared to the gravitational term, so the magnetic field has only a small effect on the radial outflow. However, the magnetic force terms in the azimuthal direction are crucially important in determining the rotational component of the wind's velocity.

Start by assuming a steady-state axisymmetric model. In the equatorial plane of the sun, we assume that the magnetic field has no component perpendicular to the plane. The magnetic field in the plane is

$$\vec{B} = B_r(r)\hat{e}_r + B_\phi(r)\hat{e}_\phi \quad (11.20)$$

and the wind velocity is

$$\vec{u} = u_r(r)\hat{e}_r + u_\phi(r)\hat{e}_\phi . \quad (11.21)$$

The electric field in the ionized wind is  $\vec{E} = -\vec{u} \times \vec{B}/c$ , and the steady-state Maxwell's equation  $\vec{\nabla} \times \vec{E} = 0$  reduces to the equation

$$\frac{1}{r} \frac{d}{dr} [r(u_r B_\phi - u_\phi B_r)] = 0 . \quad (11.22)$$

Integrating, we find

$$r(u_r B_\phi - u_\phi B_r) = \text{constant} = -\Omega_* r^2 B_r, \quad (11.23)$$

where  $\Omega_*$  is the rotation velocity of the sun.

Since the model is axisymmetric, the equation for the conservation of angular momentum in the equatorial plane is

$$\frac{d}{dr}(ru_\phi) = \frac{B_r r^2}{M} \frac{d}{dr}(rB_\phi). \quad (11.24)$$

Since  $B_r r^2 = \text{constant}$ , we may instantly perform the integration to find that

$$r \left( u_\phi - \frac{r^2 B_r}{M} B_\phi \right) = L = \text{constant}. \quad (11.25)$$

The constant  $L$  is the specific angular momentum carried away by the solar wind. The first term in the above equation is the specific angular momentum carried by the motions of the gas; the second term represents the angular momentum carried by the magnetic field.

It is convenient to define the radial Alfvénic Mach number,  $M_A \equiv u_r/v_A$ , where  $v_A$  is the Alfvén velocity,

$$v_A = \left( \frac{B_r^2}{4\pi\rho} \right)^{1/2}. \quad (11.26)$$

At the base of the corona,  $v_A \approx 300 \text{ km s}^{-1}$ ; at 1 AU, the Alfvén velocity has fallen to  $v_A \approx 40 \text{ km s}^{-1}$ . The azimuthal velocity of the solar wind, in terms of  $\Omega_*$ ,  $M_A$ , and  $L$ , is

$$u_\phi = \Omega_* r \frac{LM_A^2 r^{-2} \Omega_*^{-1} - 1}{M_A^2 - 1}. \quad (11.27)$$

At the base of the corona, where the radial velocity  $u_r$  is still small, the Alfvénic Mach number is much smaller than one. At 1 AU, the Alfvénic Mach number is  $M_A \approx 10$ . Thus, somewhere between the sun and the earth, there exists an Alfvénic critical radius at which  $M_A = 1$ . Let the radius at this point be  $r_A$  and the radial velocity be  $u_A$ . Since the denominator in the equation for  $u_\phi$  vanishes at  $r_A$ , the numerator must vanish, too. Thus,  $L = \Omega_* r_A^2$ . The angular momentum per unit mass within the solar wind can

be computed as if there were solid body rotation, at an angular velocity  $\Omega_*$ , out as far as the Alfvén radius  $r_A$ .

Because the angular momentum of the corona is being lost to the solar wind, the sun is slowly being spun down. The rate at which the sun is losing angular momentum is

$$\dot{J} = -L\dot{M} = -\Omega_* r_A^2 \dot{M}. \quad (11.28)$$

For typical models of the magnetized solar wind, the Alfvén radius is  $r_A = 1.7 \times 10^{12} \text{ cm} = 24 R_\odot = 0.11 \text{ AU}$ . With the sun's angular velocity of  $\Omega_* = 3 \times 10^{-6} \text{ sec}^{-1}$  and mass loss rate of  $\dot{M} = 2 \times 10^{-14} M_\odot \text{ yr}^{-1}$ , this implies

$$\dot{J} = -1 \times 10^{31} \text{ g cm}^2 \text{ sec}^{-2}. \quad (11.29)$$

Since the sun's total angular momentum is  $J = 1.6 \times 10^{48} \text{ g cm}^2 \text{ sec}^{-1}$ , the time scale over which the sun will be spun down is

$$t_J = -J/\dot{J} \approx 2 \times 10^{17} \text{ sec} \approx 5 \times 10^9 \text{ yr}. \quad (11.30)$$

This is a time comparable to the age of the sun, in contrast to the mass loss time scale  $t_M = M_\odot/\dot{M} \approx 5 \times 10^{13} \text{ yr}$ . Although the solar wind will not significantly affect the total mass of the sun, it will affect the total angular momentum of the sun.

The angular velocity of the wind may be rewritten as

$$u_\phi = \frac{\Omega_* r u_A - u_r}{u_A (1 - M_A^2)} \quad (11.31)$$

and the azimuthal component of the magnetic field is

$$B_\phi = -B_r \frac{\Omega_* r}{u_A} \frac{r_A^2 - r^2}{r_A^2 (1 - M_A^2)}. \quad (11.32)$$

If we knew the velocity  $u_r$  as a function of  $r$ , we could solve these equations to find  $u_\phi(r)$  and  $B_\phi(r)$ . In practice, since the presence of rotation and magnetic fields have little effect on the radial motions, we can use the standard Parker wind solution to find  $u_r(r)$ . Well within the Alfvén radius, the azimuthal velocity has the value  $u_\phi = \Omega_* r$ . Far outside the Alfvén radius,  $u_r$  is nearly constant, and the Alfvénic Mach number increases as  $M_A \propto r$ . Thus,  $u_\phi$  decreases at the rate  $u_\phi \propto r^{-1}$ . The maximum azimuthal velocity  $u_\phi$  occurs in

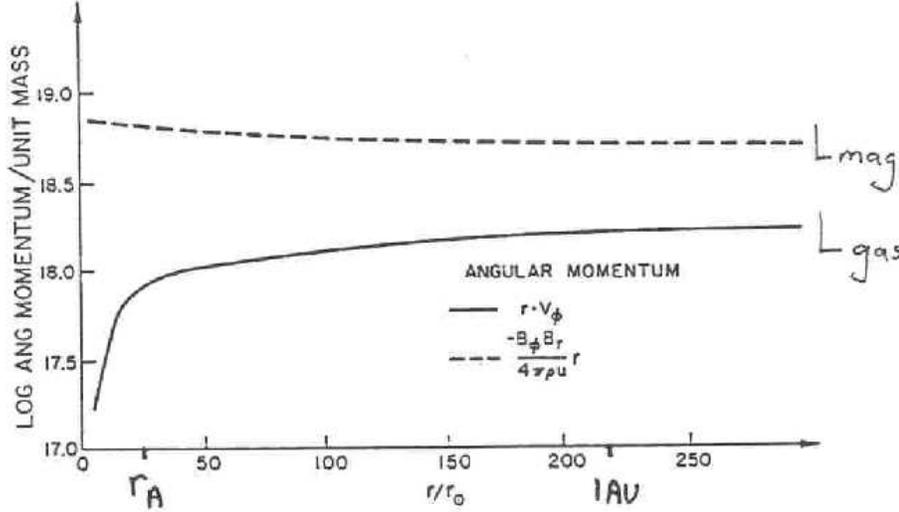


Figure 11.4: The angular momentum content of the gas in the solar wind (solid line) compared to the angular momentum content of the magnetic field (dashed line).

the vicinity of the Alfvén radius. The azimuthal component of the magnetic field decreases at the rate  $B_\phi \propto r^{-1}$ . Since  $B_r \propto r^{-2}$ , at large radii, the magnetic field will be mainly azimuthal; at small radii, the field will be mainly radial.

At a radius  $r$ , the gas in the solar wind has a specific angular momentum

$$L_{\text{gas}} = u_\phi r = \frac{\Omega_* r^2 u_A - u_r}{u_A (1 - M_A^2)}, \quad (11.33)$$

plotted as the solid line in Figure 11.4. The magnetic field carries the specific angular momentum

$$L_{\text{mag}} = L - L_{\text{gas}} = \Omega_* \frac{r_A^2 - r^2}{1 - M_A^2}, \quad (11.34)$$

which is shown as the dashed line in Figure 11.4. The ratio of angular momentum in the gas to angular momentum in the magnetic field is, at small radii,

$$L_{\text{gas}}/L_{\text{mag}} \approx \left(\frac{r}{r_A}\right)^2 \sim 2 \times 10^{-3} \left(\frac{r}{R_\odot}\right)^2. \quad (11.35)$$

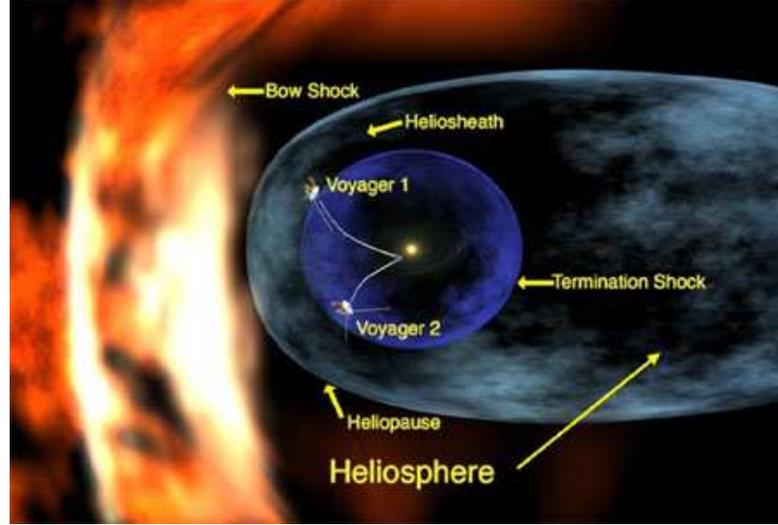


Figure 11.5: The termination shock, heliopause, and bow shock surrounding the solar system.

At large radii, where  $r \gg r_A$ ,

$$L_{\text{gas}}/L_{\text{mag}} \approx \frac{u_r}{u_A} - 1 \sim 2 \left( \frac{u_r}{700 \text{ km s}^{-1}} \right) - 1. \quad (11.36)$$

Inside the Alfvén radius, the magnetic field carries most of the angular momentum of the solar wind; outside the Alfvén radius, it shares the job with the gas.

The solar wind is shocked and decelerated when its kinetic energy density  $\rho u_r^2/2$  is comparable to the ambient pressure  $P_i$  of the interstellar medium. This happens at a radius

$$r_s \approx 140 \text{ AU} \left( \frac{\dot{M}}{10^{-14} M_{\odot} \text{ yr}^{-1}} \right)^{1/2} \left( \frac{u_r}{700 \text{ km s}^{-1}} \right)^{1/2}, \quad (11.37)$$

if the pressure of the ISM is taken to be  $P_i \approx 3 \times 10^{-13} \text{ dyne cm}^{-2}$ . The spacecraft *Voyager 1* went through this **termination shock** in 2004 December, when it was 94 AU from the Sun (Figure 11.5). *Voyager 2* went through the termination shock in 2006 May, when it was only 76 AU from the Sun; this gives a idea of the non-sphericity of the termination shock.