Chapter 12

Winds from Hot & Cool Stars

Hot luminous stars, such as O and B supergiants, are known to have stellar winds. In the UV, they display P Cygni line profiles.¹ P Cygni profiles show absorption at short wavelengths and emission at longer wavelengths, as seen in the lower panel of Figure 12.1. This asymmetric absorption/emission profile is the characteristic signature of an expanding stellar atmosphere. The absorption comes from material between us and the photosphere of the star (region 5 in the upper panel of Figure 12.1). The emission comes from material in regions 1, 2, 3, and 4. The width of the P Cygni line tells us the terminal velocity $u_\infty$ of the expanding gas. The depth of the absorption component tells us the column density of absorbing matter along the line of sight to the star. From this, we can use a model of the expanding atmosphere to deduce the total mass of the gas surrounding the star. The P Cygni lines of OB supergiants show expansion velocities of $\sim 2000 \text{ km s}^{-1}$, and mass loss rates of $\sim 10^{-6} \text{ M}_\odot \text{ yr}^{-1}$. The hottest stars can have maximum wind velocities of as much as $4000 \text{ km s}^{-1}$.

Mass loss from hot luminous stars cannot be explained by gas pressure gradients like those that drive the solar wind. Among the P Cygni lines that are observed in OB supergiants are those of CIV and SiIV. These ions are present at a temperature of $\sim 3 \times 10^5 \text{ K}$; if the gas were any hotter, they would be collisionally ionized to higher ionization states. In a hot coronal wind driven by gas pressure, the maximum observed velocity can be only a few times the sound speed, which is $a \sim 70 \text{ km s}^{-1}$ at $T \sim 3 \times 10^5 \text{ K}$. Thus,

¹P Cygni itself is a variable blue hypergiant (B1 Ia); it is visible to the naked eye, despite being some 2 kpc from the Earth. It's an example of the type of hot, luminous star that produces a fast stellar wind with a high mass-loss rate.
Figure 12.1: Formation of a P Cygni line profile.
we need some additional force to explain the very high wind velocities seen in OB supergiants.

Winds in hot luminous stars are driven by radiation pressure. The stellar winds emerging from OB supergiants have numerous resonance lines in the UV, which coincidentally is where the continuum radiation of an OB star has its maximum. The winds from OB stars are thus referred to as **line-driven** winds, since the opacity of the accelerated material is provided by absorption lines. A rough estimate of the mass-loss rate in a radiatively driven wind can be computed by assuming that each photon emitted by the star transfers its momentum of $h\nu/c$ to a gas particle in the wind. The rate at which the star loses momentum is $L/c$, where $L$ is the radiative luminosity. The rate at which the wind carries away the momentum is $\dot{M}u_\infty$, where $u_\infty$ is the asymptotic wind velocity. By setting the two rates equal to each other, we estimate a mass loss of

$$\dot{M}_{\text{est}} = \frac{L}{u_\infty c} \quad (12.1)$$

and a corresponding kinetic energy luminosity of

$$\frac{1}{2} \dot{M}_{\text{est}} u_\infty^2 = \frac{u_\infty}{2c} L \quad (12.2)$$

For a luminous O or B star, this estimated mass loss rate is $\dot{M}_{\text{est}} \sim 10^{-6} \rightarrow 10^{-5} M_\odot \text{ yr}^{-1}$. This is in the right range to account for the observed mass loss from OB stars. The kinetic energy luminosity is less than 1% of the radiation luminosity $L$. If each photon from the star undergoes multiple scatterings, then the mass loss rate can be several times $\dot{M}_{\text{est}}$.

Even if the circumstellar gas had no resonance lines, there would still be radiation pressure on the ionized gas as a result of Thomson scattering. A star of luminosity $L$ will exert an outward radial force of magnitude $f = (\sigma_T L)/(4\pi r^2 c)$ on every electron-proton pair (where $\sigma_T = 6.7 \times 10^{-25} \text{ cm}^2$ is the Thomson cross section). The ratio of the outward acceleration due to Thomson scattering, to the inward acceleration due to gravitation, is

$$\Gamma = \frac{\sigma_T L}{4\pi G M m_p c} = \frac{3.2 \times 10^{-5}}{M/L}, \quad (12.3)$$

where the mass-to-light ratio $M/L$ is in solar units. The dimensionless number $\Gamma$ is the ratio of a star’s luminosity $L$ to its Eddington luminosity $L_{\text{Edd}} = 4\pi G M m_p c/\sigma_T$. Thomson scattering alone will not be enough to
drive the stellar winds. For a very luminous O star, with \( L \approx 10^6 L_\odot \) and \( M \approx 60 M_\odot \), you find \( \Gamma \approx 1/2 \); for less luminous stars, \( \Gamma \) will be even smaller.

A single line will provide an acceleration

\[
g_L = \frac{\kappa_L}{c} \int_{\nu=0}^{\infty} \pi F_\nu(\nu) \phi(\nu) d\nu ,
\]

where \( \kappa_L \) is the line opacity (in units of \( \text{cm}^2 \text{g}^{-1} \)), \( \pi F_\nu \) is the flux from the central star at frequency \( \nu \) (in units of \( \text{erg} \text{sec}^{-1} \text{cm}^{-2} \text{Hz}^{-1} \)) and \( \phi(\nu) \) is the line profile function.

A complication is added by the fact that the gas of the stellar wind is being steadily accelerated away from the star. Thus, the radiation that each bit of gas sees from the central star will be more and more redshifted. Suppose that the gas has a strong line at a frequency \( \nu_0 \), as measured in the rest frame. A photon emitted by the star with a frequency \( \nu > \nu_0 \) will be scattered by the shell of gas that has a velocity

\[
u(r) = c \frac{\nu - \nu_0}{\nu_0}.
\]

The stellar wind will be able to scatter photons with frequencies between \( \nu_0 \) and \( \nu_0 + \delta\nu \), where \( \delta\nu = \nu_0(u_\infty/c) \). Photons with a frequency \( \nu_0 \) will be scattered near the surface of the star; photons with a frequency \( \nu_0 + \delta\nu \) will be scattered far from the star, where the expansion velocity has reached its asymptotic value of \( u_\infty \).

That maximum mass loss rate that can be produced by a single strong line is found by setting the mass momentum flux equal to the radiation momentum flux in the frequency range \( \nu_0 \rightarrow \nu_0 + \delta\nu \):

\[
\dot{M} u_\infty = \frac{L_\nu \delta\nu}{c} = \frac{L_\nu \nu_0}{c^2} u_\infty .
\]

The quantity \( L_\nu \delta\nu \) is the stellar luminosity in the frequency range \( \nu \rightarrow \nu + \delta\nu \). If \( \nu_0 \) is near the peak of the star’s energy distribution, then \( \dot{M} \sim L/c^2 \). This rate is smaller by a factor \( u_\infty/c \) than the mass loss rate \( \dot{M}_{\text{est}} \) that occurs if all photons are scattered or absorbed, rather than those in the restricted frequency range \( \nu_0 \rightarrow \nu_0(1 + u_\infty/c) \). Even for the fastest stellar winds, the ratio \( u_\infty/c \) is only 0.01. If a line-driven wind is to effectively convert the momentum of photons into the momentum of gas, it must have many strong lines in the frequency range where the star emits most of its radiation.
Suppose that a radiative flux $F_\nu$ is incident on the inner side of a thin gas shell. If the gas scatters photons with a frequency $\nu_0$, then the observed resonance line will have a Doppler width $\Delta \nu_D = \nu_0 u_t / c$, where $u_t = (kT/m)^{1/2}$ is the thermal velocity within the shell. The radiative acceleration of the shell is

$$g_L = \kappa_L \frac{\pi F_\nu \Delta \nu_D}{c} \frac{1 - e^{-\tau_L}}{\tau_L}.$$  \hspace{1cm} (12.7)

In the above equation, $\tau_L$ is the effective optical depth of the shell, and is given approximately by the equation

$$\tau_L = \frac{\kappa_L \rho(r) u_t(r) \left( \frac{du}{dr} \right)^{-1}}{c}.$$  \hspace{1cm} (12.8)

For strong lines, where $\tau_L \gg 1$, the acceleration is

$$g_L \approx \frac{\pi F_\nu \nu_0}{c} \frac{du}{\rho c dr} ,$$  \hspace{1cm} (12.9)

while for weak lines, where $\tau_L \ll 1$,

$$g_L \approx \frac{\pi F_\nu \nu_0 \kappa_L u_t}{c}.$$  \hspace{1cm} (12.10)

In real stellar winds, the total line acceleration,

$$g_L = \sum_i g_{L,i} ,$$  \hspace{1cm} (12.11)

will be a sum over weak and strong lines. It is customary (following Cas- tor, Abbott, & Klein 1975, ApJ, 195, 157), to parameterize the total line acceleration in a spherically symmetric wind as

$$g_L = \left( \frac{GM}{r^2} \Gamma \right) k \left( \frac{1}{\kappa \rho u_t} \frac{du}{dr} \right)^{\alpha} ,$$  \hspace{1cm} (12.12)

where the force constant $k$ and the slope $\alpha$ are found by an empirical fit to the observed line strengths in stellar atmospheres. (The parameter $\kappa = \sigma_T / m_p = 0.40 \text{ cm}^2 \text{ g}^{-1}$ is the Thomson scattering opacity.) If all the observed lines are strong, $\alpha = 1$; if all the lines are weak, $\alpha = 0$. An O4 star has $k \approx 1/30$ and $\alpha \approx 0.7$. 
With this parametric form for the line acceleration, the isothermal Bondi equation takes the modified form

\[
\frac{1}{2} \frac{d(u^2)}{dr}(1 - \frac{a_0^2}{u^2}) = -\frac{GM}{r^2} \left[ 1 - \frac{2a_0^2 r}{GM} - \Gamma - \Gamma k t^{-\alpha} \right], \tag{12.13}
\]

where

\[
t = \left( \frac{\kappa u_t \dot{M}}{4\pi} \right) \left( \frac{r^2 d(u^2)}{2 dr} \right)^{-1}. \tag{12.14}
\]

In the absence of line opacity, the solution to this equation is a Parker wind with a sonic radius at

\[
r_s = \frac{GM(1 - \Gamma)}{2a_0^2}. \tag{12.15}
\]

Once the line opacity is added, the sonic radius is no longer a critical point. There is, however, a unique solution which has a smooth transition from subsonic flow near the star to supersonic flow far from the star. The terminal velocity for this model has the value

\[
u_\infty = \left( \frac{\alpha}{1 - \alpha} \right)^{1/2} v_{\text{esc}}, \tag{12.16}
\]

where

\[
v_{\text{esc}} = \left[ \frac{2GM(1 - \Gamma)}{R_*} \right]^{1/2}. \tag{12.17}
\]

The observed values of \(u_\infty/v_{\text{esc}}\) for a sample of hot, luminous stars are plotted in Figure 12.2. The O stars are observed to have \(u_\infty/v_{\text{esc}} \approx 3\), suggesting a value of \(\alpha \approx 0.9\) for these stars. Late B stars, by contrast, have \(u_\infty/v_{\text{esc}} \approx 1\), yielding \(\alpha \approx 0.5\). The mass loss rate for a line-driven wind is

\[
\dot{M} = \frac{L}{u_t c} k^{1/\alpha} \alpha (1 - \alpha)^{(1-\alpha)/\alpha} \left( \frac{\Gamma}{1 - \Gamma} \right)^{(1-\alpha)/\alpha}. \tag{12.18}
\]

This leads to a luminosity dependence \(\dot{M} \propto L^{1/\alpha}\). If all the lines are optically thick, then \(\dot{M} \propto L\); if all the lines are optically thin, then there will be a very steep dependence of \(\dot{M}\) on \(L\).

An O5 star has a mass \(M = 60 M_\odot\), a luminosity \(L = 10^6 L_\odot\), and a radius \(R_* = 14 R_\odot\). The effective temperature is then 49,000 K. Castor, Abbott, and Klein used their parametric model, with \(k = 1/30\) and \(\alpha = 0.7\), to deduce
Figure 12.2: The ratio of terminal wind velocity to escape speed as a function of spectral type for stars from O4 to A2.
a terminal velocity \( u_\infty = 1500 \text{ km s}^{-1} \) for such a star, and a mass loss rate of \( \dot{M} = 7 \times 10^{-6} \text{ M}_\odot \text{ yr}^{-1} \). This is in good agreement with observations of O stars. The time scale for mass loss in such a star is \( t_M = M/\dot{M} = 1 \times 10^7 \text{ yr} \), which is only three times the main sequence lifetime of an O5 star.

The terminal velocities deduced for cool luminous stars range from \( u_\infty \sim 10 \text{ km s}^{-1} \) for M supergiants to \( u_\infty \sim 75 \text{ km s}^{-1} \) for K giants. These velocities are smaller than the corresponding escape velocities \( v_{\text{esc}} \) from the stellar surface. An approximate fit, found empirically, is

\[
u_\infty \sim \frac{v_{\text{esc}}^2}{1000 \text{ km s}^{-1}}.
\]

The deduced mass loss rate for cool luminous stars lies in the approximate range \( 10^{-8} \text{ M}_\odot \text{ yr}^{-1} \rightarrow 10^{-5} \text{ M}_\odot \text{ yr}^{-1} \).

Winds in cool K and M stars cannot be driven by pressure gradients, since cool stars with winds lack hot, extended coronas. Thomson scattering is not enough to drive the winds of cool stars. An M supergiant, with \( L = 3 \times 10^4 \text{ L}_\odot \) and \( M = 20 \text{ M}_\odot \), will have \( \Gamma = L/L_{\text{Edd}} = 3.2 \times 10^{-5}(M/L)^{-1} = 0.05 \). A K giant, with \( L = 130 \text{ L}_\odot \) and \( M = 4 \text{ M}_\odot \), will have \( \Gamma = 0.001 \). The radiative acceleration from lines is also small; the continuum of cool stars peaks in infrared or red, where there are few resonance lines in the atmosphere. However, in cool giants and supergiants, there is an additional source of opacity. Cool giants and supergiants are observed to emit an excess of radiation in the infrared. This infrared excess can be attributed to the formation of dust grains in the cool, extended stellar envelope; dust condensation requires temperatures lower than \( T \sim 1000 \text{ K} \).

The infrared opacity of the atmosphere increases very rapidly at the radius where the dust condenses out. The grains, once they form, are accelerated outward as the result of the momentum they gain by absorbing photons. Some of the momentum of the grains is transferred to the gas by collisions. As the grains are driven outward, they thus drag the gas along with them. When the dust grains are accelerated to high velocities, however, they begin to be destroyed by ‘sputtering’. The acceleration ceases, and the terminal wind velocity stays at the relatively low value of \( \sim 20 - 50 \text{ km s}^{-1} \). If dust particles were indestructible, the wind could be accelerated to velocities of \( \sim 100 \text{ km s}^{-1} \) or more. The radiative acceleration on the dust grains is greater than the gravitational acceleration when

\[
\kappa_d > \frac{4\pi GMc}{L} = 1.3 \times 10^4 \text{ cm}^2 \text{ g}^{-1}\left(\frac{M}{L}\right),
\]

(12.20)
where $\kappa_d$ is the frequency-averaged opacity of the dust, and the mass-to-light ratio $M/L$ is in solar units. An M supergiant, with $M/L = 7 \times 10^{-4} \, M_\odot / L_\odot$, will require a dust opacity of $\kappa_d > 9 \, \text{cm}^2 \, \text{g}^{-1}$ in order to have a dust-driven wind; a K giant, with $M/L = 0.03 \, M_\odot / L_\odot$, will require $\kappa_d > 400 \, \text{cm}^2 \, \text{g}^{-1}$.

Radiative pressure on dust cannot be the sole mechanism driving winds from cool luminous stars, since dust can only exist in a relatively cool environment. Infrared observations of Betelgeuse (an M supergiant) show that less than 20% of the emission from dust comes from within a distance $12R_*$ of the star. For K stars, with their higher effective temperatures, the problem of forming dust close to the star’s surface is even greater. If dust forms at 10 times the stellar radius, then some other mechanism must lift gas 90% of the way out of the star’s gravitational well before it even encounters the dust.

Winds from cool, luminous stars might be driven by Alfvén waves propagating outward in the atmosphere of a magnetized star. Large amplitude Alfvén waves ($\delta B \sim B$) are observed in the sun’s atmosphere, so it is not implausible that such waves may propagate through other stellar atmospheres as well. The rate at which Alfvén waves deposit momentum in the star’s atmosphere depends on whether the waves are adiabatic (undamped) or dissipational (damped). Hartman & MacGregor, 1980 (ApJ, 242, 260) examined models of Alfvén-wave driven winds. For a supergiant with $M = 16 \, M_\odot$, $R = 400 \, R_\odot$ and a surface magnetic field of $B = 10 \, \text{G}$, the predicted terminal velocity is $u_\infty = 400 \, \text{km} \, \text{s}^{-1}$ if the Alfvén waves are undamped. If the Alfvén waves are heavily damped, then all of the wave energy goes into heating the base of the atmosphere. If, however, the damping length of the Alfvén waves is comparable to the radius of the star, then the observations can be matched. For a $16 \, M_\odot$, $400 \, R_\odot$, $10 \, \text{G}$ supergiant, with an Alfvén damping length of $L = R_\star = 400 \, R_\odot$, the terminal velocity of the wind is $u_\infty = 50 \, \text{km} \, \text{s}^{-1}$, and the mass loss rate is $\dot{M} = 5 \times 10^{-7} \, M_\odot \, \text{yr}^{-1}$, in agreement with observed values.

Another possible mechanism for driving winds from cool, luminous stars is pulsationally driven, radially expanding shock waves in the stellar atmosphere. Many M giants and supergiants are observed to be variable, with long periods. For instance, Mira (an M giant) has a period of 330 days, during which its luminosity varies by 8 magnitudes. The variability in luminosity is due to the radial pulsation of the star. The pulsating star can be thought of as a spherically symmetric piston, driving periodic shock waves into the stellar atmosphere. Every year or so, before the gas has a chance to settle down into hydrostatic equilibrium again, it is hit by another shock, and yet
more momentum and energy is added to the gas. Eventually, the outer layers of the star are unbound, and flow outward in a wind. The properties of a shock-driven wind, as illustrated in Figure 12.3, depend on whether the gas is adiabatic or isothermal. A model star with $M = M_\odot$, $L = 10^4 L_\odot$ and pulsational period $P = 373 days$ will have a mass loss rate of $\dot{M} = 0.02 M_\odot yr^{-1}$ if the gas is adiabatic, and a mass loss rate of $\dot{M} = 10^{-12} M_\odot yr^{-1}$ if the gas is isothermal. The main difference in the two types of flow is that the adiabatic case has much higher densities at large radii. A combination of isothermal shock-driven flow (effective at small radii) and dust-driven flow (effective at large radii) gives a mass loss rate of $\dot{M} = 3 \times 10^{-7}$ and a terminal velocity $u_\infty = 7 \text{ km s}^{-1}$, which are of the right magnitude for mass loss from a Mira variable.