Ast 825: Radiative Gas Dynamics

Problem Set 1

Due Thursday, January 22

1) Let's start simply, with a hydrostatics problem, rather than a hydrodynamics problem. The Earth's atmosphere may be approximated as a static, plane-parallel atmosphere, with a uniform gravitational acceleration g. To make things easier still, let's assume that the Earth's atmosphere is made of an ideal gas.

a) If $P(z)$ and $T(z)$ are the pressure and temperature at a height z above sea level, show that

$$\frac{1}{P}\frac{dP}{dz} = -\frac{gm}{kT} \ ,$$

where $g = 980\,\mathrm{cm\,s^{-2}}$ is the magnitude of the gravitational acceleration and $m = 4.8 \times 10^{-23}\,\mathrm{g}$ is the mean molecular mass of the atmosphere.

b) Assume that the decrease in pressure with height is due to an *adiabatic* expansion of the gas; in this case, the atmosphere follows a polytropic equation of state, with $P \propto \rho^\gamma$, where γ is the adiabatic index. What are the resulting pressure, density, and temperature as a function of height, given their values P_0, ρ_0, and T_0 at sea level? Give numerical results, adopting reasonable values for the sea-level temperature and pressure.

c) The results of section (1b) are nonsense in the limit of large z. [Hint: if your answer to section (1b) is not nonsensical in this limit, you've done something wrong.] What false assumptions did we make that led to this nonsensical result?

d) Now assume that the atmosphere is not adiabatic, but *isothermal* (that is, the heating and cooling processes in the atmosphere keep it at a uniform temperature $T = T_0$.) What are $P(z)$ and $\rho(z)$ in this case? Again, give numerical results.

2) Let's now look at a steady-state hydrodynamics problem. ("Steady-state" means that u, ρ, P, and so forth have no explicit time dependence.)

a) A star of mass M_* and radius R_* emits a spherically symmetric wind at a constant rate $\dot{M}$. The wind is expanding freely into a surrounding vacuum. Write down the Euler equations (equations 2.3, 2.4, and 2.5 of the notes) for this spherically symmetric, nonrotating, steady-state system.

b) Assume that the pressure gradient and self-gravity of the gas are negligible compared to the gravitational acceleration $g = GM_*/r^2$ of the central star. Find the radial velocity $u(r)$ of the wind in terms of M_*, R_*, and $u_* \equiv u(R_*)$.

c) Using the continuity equation and the wind velocity, $u(r)$, find the density $\rho(r)$ of the stellar wind.

d) At what radius does the wind's self-gravity become comparable to the gravity of the central star? (An approximate estimate is all right.)

3) Now let's start looking at simple shocks traveling through different media.

a) A normal plane-parallel nonradiative shock moves with Mach number $M_1 = 1.5$ through the earth's atmosphere ($\rho_1 = 1.2 \times 10^{-3}\,\mathrm{g\,cm^{-3}}$, $P_1 = 1.0 \times 10^{6}\,\mathrm{dyne\,cm^{-2}}$, $\gamma = 7/5$). What are ρ_2, P_2, and T_2 immediately after passage of the shock? What is the relative velocity, $u_1 - u_2$, of the pre-shock and post-shock gas?

b) A normal plane-parallel nonradiative shock moves with Mach number $M_1 = 1.5$ through the warm neutral ISM ($\rho_1 \approx 10^{-24}\,\mathrm{g\,cm^{-3}}$, $P_1 \approx 4 \times 10^{-13}\,\mathrm{dyne\,cm^{-2}}$, $\gamma = 5/3$). What are ρ_2, P_2, and T_2 immediately after passage of the shock? What is the relative velocity, $u_1 - u_2$, of the pre-shock and post-shock gas?

4) Shocks have interesting implications for star formation. Let's see what a shock does to the Jeans mass m_J of the interstellar medium.

a) A plane-parallel *nonradiative* shock, with Mach number M, passes through a gaseous medium. What is the ratio of the Jeans mass m_{J2} in the post-shock medium to the Jeans mass m_{J1} in the pre-shock medium?

b) A plane-parallel *isothermal* shock, with isothermal Mach number M_T, passes through a gaseous medium. What is the ratio of the Jeans mass m_{J3} in the post-radiative medium (far downstream of the radiative recombination layer) to the Jeans mass m_{J1} in the pre-shock medium?

5) For a very weak normal shock ($M_1 = 1 + \epsilon$, with $\epsilon \ll 1$), find the change in specific entropy, $s_2 - s_1$, across the shock layer, expressed as a function of ϵ.