Ast 825: Radiative Gas Dynamics

Problem Set 2

Due Thursday, February 5

1) It's what you've been waiting for: a radiative MHD shock! Suppose that a radiative, steady-state, plane parallel shock has a bulk velocity $\vec{u}$ perpendicular to the shock front and magnetic field $\vec{B}$ parallel to the shock front. Let the subscript '1' designate the properties of the gas upstream of the shock transition layer; let '2' designate properties immediately downstream of the shock transition layer; let '3' designate the properties far downstream, beyond the radiative relaxation layer. The Rankine-Hugoniot jump conditions for this radiative MHD shock are:

$$\rho_1 u_1 = \rho_2 u_2 = \rho_3 u_3$$

$$\rho_1 u_1^2 + P_1 + \frac{B_1^2}{8\pi} = \rho_2 u_2^2 + P_2 + \frac{B_2^2}{8\pi} = \rho_3 u_3^2 + P_3 + \frac{B_3^2}{8\pi}$$

$$\frac{\gamma}{\gamma - 1}\frac{P_1}{\rho_1} + \frac{1}{2}u_1^2 + \frac{B_1^2}{4\pi\rho_1} = \frac{\gamma}{\gamma - 1}\frac{P_2}{\rho_2} + \frac{1}{2}u_2^2 + \frac{B_2^2}{4\pi\rho_2}$$

$$B_1 u_1 = B_2 u_2 = B_3 u_3$$

a) Find the value for the density ratio ρ_2/ρ_1, in terms of the adiabatic index γ, the upstream Mach number $M_1 \equiv u_1/a_1$, and the ratio of pressures $\alpha_1 \equiv B_1^2/(8\pi P_1)$.

b) Suppose the shock is isothermal, so that $T_3 = T_1$. Find the value for the density ratio ρ_3/ρ_1, in terms of γ, M_1, and α_1.

c) For a shock with $M_1 = 5$ and $\gamma = 5/3$, plot ρ_2/ρ_1 and ρ_3/ρ_1 as a function of α_1 in the range $0 \leq \alpha_1 \leq 20$.

2) A lightning bolt can be approximated as a linear explosion, releasing an energy per unit length ϵ in a gaseous medium of mass density ρ.

a) If the shock created by the lightning is a cylindrical blastwave (that is, if radiative energy losses are negligible), what is the radius of the expanding cylindrical shell as a function of time? (Note that there will be a dimensionless factor of order unity in your solution, comparable to ξ_0 for a spherical blastwave – it is not necessary to compute the exact value of this number. Life is too short.)

b) If $\epsilon = 2 \times 10^{12}\,\mathrm{erg\,cm^{-1}}$, a typical value for lightning, and if $\rho = 0.001\,\mathrm{g\,cm^{-3}}$, find the radius at which the pressure behind the shock is equal to the ambient pressure $P_1 = 1 \times 10^6\,\mathrm{dyne\,cm^{-2}}$ of the atmosphere.

3) This question applies the basic principles of turbulence (which we'll be discussing on Tuesday) to a genuine astrophysical object.

Castañeda (1988, ApJS, 67, 93) has measured the turbulent line widths of emission lines in the Orion Nebula. His results for one region of the nebula are given below.

ℓ [arcsec]	Δv [km/sec]
8	1.67 ± 0.29
12	2.26 ± 0.31
16	2.94 ± 0.33
24	2.87 ± 0.46
32	3.50 ± 0.71
40	3.72 ± 0.23

If you assume the relation $\Delta v = K(\ell/1\,\mathrm{pc})^{1/3}$, what is the best-fitting value for K? (And please let me know what value you choose for the distance to the nebula). If you assume that the turbulence has a Kolmogorov spectrum, and that $T = 9200\,\mathrm{K}$ and $n = 5.0 \times 10^3\,\mathrm{cm}^{-3}$, what are the values of ϵ_d and the kinematic viscosity ν? What are the Kolmogorov length scale ℓ_K, velocity scale τ_K, and time scale u_K? On what length scale is the turbulent motion supersonic? How does this length scale compare to the size of the nebula?