

Radiative Gas Dynamics

Problem Set 3

Due Thursday, February 19

1) The atmospheres of Venus and Mars are made of carbon dioxide, which has a kinematic viscosity $\nu = 0.070 \text{ cm}^2 \text{ s}^{-1} (T/100 \text{ K})^{1/2}$. Solar energy is added to the Venusian atmosphere at a rate $\epsilon_d = 0.1 \text{ erg s}^{-1} \text{ g}^{-1}$. Solar energy is added to the Martian atmosphere at the rate $\epsilon_d = 120 \text{ erg s}^{-1} \text{ g}^{-1}$.

a) Assuming a Komogorov spectrum, what is the size ℓ_K of the smallest eddies in the atmosphere of Venus? What is ℓ_K in the atmosphere of Mars? (Yes, you will have to pick a typical temperature for the atmosphere of each planet.)

b) What is the speed $u(\ell)$ with which eddies of size ℓ swirl about in the atmosphere of Venus? What is the velocity $u(\ell)$ in the atmosphere of Mars?

2) Consider the case of spherical steady-state accretion of an isothermal gas, in which the temperature is held constant at the value T_0 .

a) Write down the value of the sonic radius r_s in terms of M_* (the mass of the central compact object) and a_0 (the isothermal sound speed of the gas). If a neutron star with mass $M_* = 1.4 M_\odot$ is accreting pure ionized hydrogen with a temperature $T_0 = 2.0 \times 10^6$ K, what is the numerical value of r_s ?

b) Write down the Bondi equation for isothermal accretion, and solve to find the relation between u and r in a transonic isothermal flow, using the boundary condition $u \rightarrow 0$ as $r \rightarrow \infty$.

c) Find the value of $u(r)$ in the limits $r \gg r_s$ and $r \ll r_s$, assuming $M_* = 1.4 M_\odot$ and $T_0 = 2.0 \times 10^6$ K.

d) Find the value of $\rho(r)$ in the limits $r \gg r_s$ and $r \ll r_s$, assuming $M_* = 1.4 M_\odot$, $T_0 = 2.0 \times 10^6$ K, and $\rho_\infty = 5 \times 10^{-27}$ g cm⁻³. What is the value of \dot{M} for the transonic flow?

3) A nova is caused by the runaway nuclear fusion of hydrogen that has accreted onto a white dwarf. Let's look at some of the energetics of a recurrent nova.

a) A white dwarf has a mass $M_* = 0.80 M_\odot$ and a radius $R_* = 0.010 R_\odot$. It is accreting mass from a companion at the rate $\dot{M} = 2.0 \times 10^{-8} M_\odot \text{ yr}^{-1}$. What is the accretion luminosity, $L_{\text{acc}} \equiv GM_*\dot{M}/R_*$, expressed in units of L_\odot ? What is L_{acc} as a fraction of the white dwarf's Eddington luminosity?

b) Every 50 years, the white dwarf has a 4-month long nova outburst. How much hydrogen does the white dwarf accrete during the course of 50 years? (Assume the donor star has the same composition as the Sun's photosphere.) How much energy is released by fusing all that hydrogen into helium? What is the time-averaged luminosity during the 4-month long outburst, expressed in units of L_\odot ? Is this greater than or less than the white dwarf's Eddington luminosity?

4) Gas dynamics is versatile! We can even apply it to Saturn's rings. The narrow F ring of Saturn is located at a distance $R_0 = 1.4 \times 10^{10}$ cm from the center of Saturn. Its width is $\delta R = 1.0 \times 10^7$ cm in the radial direction and its vertical scale height is $H = 3.0 \times 10^3$ cm. Make the simplifying assumption that the optical depth of the ring is $\tau = 6.0$ (looking perpendicular to the ring plane), and that the ring particles are uniform, elastic spheres with radius $r = 3.0$ cm and mass $m = 120$ g. (General hint: Yes, you will need to know the mass of Saturn. No, I won't care strongly about factors of π and $\sqrt{2}$.)

- a) What is σ_z , the r.m.s. velocity of ring particles perpendicular to the ring?
- b) What is λ , the mean free path for ring particles near the midplane of the ring?
- c) What is the kinematic viscosity ν of the ring? (Assume an isotropic velocity dispersion.)
- d) What is the timescale t_{vis} over which the ring will be broadened by viscous spreading?