Radiative Gas Dynamics

Problem Set 4

Due Thursday, March 5

1) Near the inner edge of an accretion disk around a neutron star, the pressure is provided by the radiation pressure:

$$P_{rad} = \frac{4\sigma_{SB}}{3c} T^4 \tag{1}$$

and the opacity is provided by Thomson scattering:

$$\kappa_R = 0.40\,\mathrm{cm}^2\,\mathrm{g}^{-1}\ . \tag{2}$$

a) Combine these two equations with the other equations for the structure of the disk:

$$\rho = \frac{\Sigma}{H} \tag{3}$$

$$H = a\left(\frac{R^3}{GM_*}\right)^{1/2} \tag{4}$$

$$a^2 = \frac{P}{\rho} \tag{5}$$

$$\frac{4\sigma_{SB}T^4}{3\tau} = \frac{3GM_*\dot{M}}{8\pi R^3}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right] \tag{6}$$

$$\tau = \Sigma\kappa_R \tag{7}$$

$$\nu\Sigma = \frac{\dot{M}}{3\pi}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right] \tag{8}$$

and

$$\nu = \alpha a H\ , \tag{9}$$

to find Σ, H, ρ, T, τ, ν, and u_R as a function of α, $R_{10} \equiv R/10^{10}\,\mathrm{cm}$, $M_1 \equiv M_*/1\,\mathrm{M}_\odot$, $\dot{M}_{16} \equiv \dot{M}/10^{16}\,\mathrm{g\,sec}^{-1}$, and $f(R) \equiv 1 - (R_*/R)^{1/2}$.

b) Suppose that the accretion disk consists of pure ionized hydrogen. At what radius R is the thermal gas pressure

$$P_{gas} = \frac{\rho k T}{m}$$

equal to the radiation pressure (thus violating the assumption made in equation 1)?

2) A compact object of mass M_* is radiating at its Eddington luminosity $L_{\rm edd}$. An astronaut wearing a fashionable, black, skin-tight, low-mass spacesuit is at rest with respect to the compact object. The astronaut is physically fit, and has a body mass index (BMI) of 20. For an individual with mass M and height h, the body mass index is given by the formula

$$\mathrm{BMI} = \left(\frac{M}{1\,\mathrm{kg}}\right)\left(\frac{h}{1\,\mathrm{m}}\right)^{-2} .$$

a) Does there exist a critical astronaut mass $M_{\rm crit}$ at which the outward radiative force on the astronaut exactly balances the inward gravitational force? If so, what is its value in kilograms? (You may assume a cylindrical astronaut whose long axis is perpendicular to the line between the astronaut and the compact object.)

b) Suppose that the fashion in spacesuits switches from black to silver. If the astronaut wears a perfectly reflective spacesuit, is there then a critical mass at which radiative force and gravity balance? If so, what is its value in kilograms?

3) The nearest supergiant to the Solar System is Betelgeuse. At a distance of 130 parsecs, it's not really close enough to do much damage to us when it becomes a supernova. However, let's see what would happen if a supernova went off at a distance $d = 13\,\mathrm{pc}$ from the Sun, rather than 130 pc.

a) A core-collapse supernova (which Betelgeuse will eventually produce) has a maximum luminosity of $L_V \approx 4 \times 10^9 L_\odot$ in the V band. What will be the corresponding apparent magnitude at $d = 13\,\mathrm{pc}$?

b) If the energy released by the supernova into the interstellar medium is $E = 10^{51}\,\mathrm{erg}$, how long will it take the expanding shock wave to reach a radius $r_{\mathrm{sh}} = 13\,\mathrm{pc}$? What will be the expansion speed u_{sh} of the shock at that time? What will be the pressure P_2 and the temperature T_2 immediately behind the shock?

c) At what distance from the Sun will the pressure P_2 immediately behind the shock wave be balanced by the kinetic energy density $\rho u_r^2/2$ of the solar wind? If this distance is greater than 1 AU, the Earth will be saved from being hammered by the shock wave – if not, interesting things will happen when the shock wave hits the Earth's magnetosphere!