11. CMB Anisotropy

Gravitational instability and structure formation

Today’s universe shows structure on scales from individual galaxies to galaxy groups and clusters up to superclusters of galaxies that can extend to 100 Mpc or more.

If the universe is smooth but not perfectly smooth, then gravity will cause structure to grow, by drawing matter into regions that start slightly above the average density (and out of regions that start slightly below the average density).

For this process to work, there must have been small-amplitude inhomogeneities present in the early universe, the “seeds” for the gravitational growth of structure.

These inhomogeneities should cause slight non-uniformities in the CMB.

In the simplest models with only baryonic matter, the predicted fluctuations in the CMB are roughly one part in $10^3$.

With non-baryonic dark matter, it is possible to lower the level of CMB anisotropies to $\sim 10^{-5}$ and remain consistent with gravitational growth of today’s structure.

Measurement of CMB anisotropy

The “dipole” anisotropy of the CMB, which is just the reflex of the earth’s peculiar velocity (which is largely due to the motion of the Milky Way), was detected in the 1970s.

The first detection of the intrinsic anisotropies of the CMB, reflecting structure in the universe at $z_{\text{rec}}$ rather than the motion of the earth today, was obtained by the COBE satellite in 1992.

The variations are only a part in $10^5$, so measuring them requires extremely high sensitivity and extremely good control of systematic errors, like contamination of the signal from other sources.

Since COBE, there have been a number of measurements of CMB anisotropy with higher resolution and sensitivity, from the ground, balloons, and satellites.

The state-of-the-art experiments are WMAP (Wilkinson Microwave Anistropy Probe), which just completed the analysis of its 9-year data set, the Planck satellite, which is more sensitive and higher resolution than WMAP, and the ground-based experiments SPT (South Pole Telescope) and ACT (Atacama Cosmology Telescope), which can get to smaller angular scales because they use larger telescopes.

Characterizing the CMB anisotropy

Suppose that we smooth a CMB map over an angular scale $\theta$. (Any real map will automatically be smoothed at some minimum angular scale determined by the diffraction limit of the telescope used to make it, and we can subsequently smooth the map over larger scales.)

We can then plot a histogram of the fractional temperature variations $\Delta T/T$ in the smoothed map and measure the root-mean-square width of this histogram.

To characterize the structure in the map, we can plot $(\Delta T/T)_{\text{rms}}$ against the smoothing scale $\theta$.

If the map consisted of randomly placed hot and cold spots, we would expect $(\Delta T/T)_{\text{rms}}$ to decline in proportion to the square-root of the sky area ($\propto 1/\theta$) because of $\sqrt{N}$ averaging of random variables.
In fact, hot and cold regions are correlated over large scales, and $(\Delta T/T)_{\text{rms}}$ declines much more slowly than $1/\theta$.

The most widely used statistical measure of structure in CMB maps is the angular power spectrum, $C_l$, derived from a decomposition of the map into spherical harmonics.

Roughly speaking, $l(l+1)C_l$ is the square of $(\Delta T/T)_{\text{rms}}$ on the scale $\theta = 200/l$ degrees.

**Physics of CMB anisotropy**

It is not surprising that if the density of the universe is slightly inhomogeneous at $z_{\text{rec}}$, then the CMB will be slightly non-uniform.

There are several effects that link the inhomogeneities of the matter distribution to the CMB anisotropies:

- **Adiabatic temperature fluctuations.** Where the density is higher, the photon temperature is hotter.
- **Doppler shifts.** The gravitational perturbations caused by the density fluctuations induce peculiar velocities, and photons are blueshifted or redshifted if they last scattered off electrons with peculiar velocities toward us or away from us, respectively.
- **Gravitational redshifts,** as the photons climb out of the dark matter potential wells, or blueshifts as they fall off of potential hills. This effect makes the photons coming out of denser regions redder (cooler temperature).

Different processes dominate on different scales, so if we can measure CMB anisotropy over a wide range of scales we can separate them to some degree.

**Acoustic oscillations**

As long as the universe is ionized, so that photons are tightly coupled to the electrons and baryons, the photons and baryons behave like a single, high-pressure fluid with a “sound speed” of approximately $c/\sqrt{3}$.

An overdense region may start to collapse under its own gravity, but eventually the buildup of pressure will halt and reverse the expansion.

If there is enough time, a perturbation can contract and re-expand several times, a cycle of “acoustic oscillations.”

Because of this effect, there is a preferred scale on which the temperature fluctuations are largest. This is the scale for which an overdense region has had just enough time to collapse to maximum compression when the universe recombines.

On very large scales there has been insufficient time for contraction to occur, and gravitational redshifts produce approximately “scale-invariant” fluctuations.

**CMB Polarization**

CMB anisotropies are polarized, i.e., they have slightly different amplitudes when measured in orthogonal polarizations.

The polarized signal is only about 10% of the full signal (which is only $10^{-5}$ in the first place), and the polarization from foreground contaminants is hard to remove, so these measurements are tough.

First detections in 2005, consistent with predictions from inflation (discussed in next section).
Roughly speaking, polarization doubles (or even triples) the information content of CMB anisotropies and better pins down what is causing anisotropy on a given scale.

These effects can produce only particular kinds of statistical patterns, known as “E-mode” polarization.

“B-mode” polarization can be produced by gravitational lensing of the E-mode pattern by clustered matter at low redshifts.

B-mode polarization can also be produced by gravity waves created in the early universe, a.k.a. “tensor fluctuations.”

Detection of B-mode polarization, especially from gravity waves, is the current holy grail of experimental CMB research.

The Planck satellite will significantly advance polarization measurements. Current and future ground-based and balloon experiments can do significantly better than Planck, while a future CMB polarization satellite would provide the ultimate in sensitivity.

CMB Anisotropy: Bottom Lines

If we specify the statistical properties of the density fluctuations present at $z_{\text{rec}}$, and the matter and energy contents of the universe ($\Omega_r, 0, \Omega_m, 0, \Omega_\Lambda, 0, H_0$, etc.), then we can predict the full pattern of CMB anisotropy.

Model predictions can be tested against measurements, and the measurements can be used to infer the properties of the primordial fluctuations and the matter and energy contents of the universe.

Changing parameters changes the predictions: for a nice illustration see http://space.mit.edu/home/tegmark/movies.html.

The interplay between gravity and pressure introduces a preferred scale, the acoustic oscillation scale, on which CMB fluctuations are strongest.

This scale, which is approximately the speed of light times the age of the universe at recombination (and which can be computed exactly once the cosmological parameters are specified), provides a “standard ruler,” whose apparent angular size can be used to measure the geometry of space.

CMB measurements have reached very high precision, but different parameters have partially degenerate effects on the predictions.

CMB measurements in concert with other data at lower redshifts provide powerful constraints on cosmological parameters and powerful tests of inflation, which is the leading theory for the origin of the primordial fluctuations.
Angular power spectrum of the CMB as measured by the Planck satellite.
For an animated version see http://space.mit.edu/home/tegmark/movies.html.