Astronomy 141 – Life in the Universe
Autumn Quarter 2008 – Prof. Gaudi
Homework #4 Solutions

These questions concern the concept of targets for searches for extrasolar planets, specifically (a) the number of targets available for these searches for stars of various types, (b) the suitability of stars of various types, and (c) the sizes of the signals for Earthlike planets for stars of various types and using various methods.

Assume that the space density of all main-sequence stars in the solar neighborhood is 0.1 stars per cubic parsec. When necessary, use the 'typical' values for the physical parameters of stars of various spectral types taken from your book or my notes.

Question 1 (10 points)

Estimate the number of A stars within 10 parsecs of the Sun.

a) 4
b) 0
c) 42
d) 419
e) 4189

The total number N of stars (of all types) within 10 parsecs is just the volume V contained by a sphere of radius 10 parsecs times the space density of stars ρ. The volume is,

$$V = \frac{4\pi}{3} d^3 = 4188.8 \text{ pc}^3$$

and so the total number of stars is,

$$N = \rho V = 0.1 \text{ pc}^{-3} \times 4188.8 \text{ pc}^3 = 419$$

The fraction of all stars that are A-type is given in Table 11.1 of your book, and is 1%. Therefore, the number of A stars within 10 parsecs of the Sun is $0.01 \times N = 0.01 \times 419 \sim 4$.

Answer a) 4

Question 2 (10 points)

Estimate the number of G stars within 10 parsecs of the Sun.

a) 2932
b) 293
c) 1
d) 7
e) 29

The fraction of all stars that are G-type is 7% according to Table 11.1 of your book. Therefore, the number of G stars within 10 parsecs of the Sun is $0.07 \times N = 0.07 \times 419 \sim 29$.

Answer e) 29

Question 3 (10 points)

Estimate the number of M stars within 5 parsecs of the Sun.

a) 1
b) 393
c) 39
d) 314
e) 5

The total number of stars within 5 parsecs of the Sun is,

$$N = \rho V = 0.1\ \mathrm{pc}^{-3} \times \frac{4\pi}{3}(5\ \mathrm{pc})^3 = 52\,.$$

Roughly 75% of all stars are M stars, and therefore the number within 5 parsecs of the Sun is $0.75 \times 52 \sim 39$.

Answer c) 39

Question 4 (25 points)

Assume that it takes 1.5 Gyr for intelligent life to evolve. Further assume that the Sun has a lifetime of 10 Gyr, and the lifetime of a main sequence star is proportional to M^{-3}, where M is the mass of the star. What kinds of main sequence stars should *not* be targeted for searches for intelligent life?

a) Stars with mass less than about 5.5 times the mass of the Sun.
b) Stars with mass less than about 1.9 times the mass of the Sun.

c) Stars with mass greater than about 1.9 times the mass of the Sun.
d) Stars with mass greater than about 0.53 times the mass of the Sun
e) Stars with mass less than about 6.7 times the mass of the Sun.

If the lifetime of main sequence stars is proportional to M^{-3}, then we can write the lifetime relative to the Sun:

$$Lifetime_{Star} = Lifetime_{Sun}\left(\frac{M}{M_{Sun}}\right)^{-3}.$$

We can use this to determine the main sequence mass for which the lifetime is 1.5 Gyr. Solving this equation for the mass of the main-sequence star relative to the Sun:

$$M = M_{Sun}\left(\frac{Lifetime_{Star}}{Lifetime_{Sun}}\right)^{-1/3} = M_{Sun}\left(\frac{1.5\text{Gyr}}{10\text{Gyr}}\right)^{-1/3} = (0.15)^{-1/3}M_{Sun} \approx 1.9M_{Sun}.$$

Since more massive stars have shorter lifetimes, we conclude that stars more massive than about 1.9 times the mass of the Sun will have lifetimes that are too short to allow intelligent life to evolve.

Answer c) Stars with mass greater than about 1.9 times the mass of the Sun.

For the following three questions, pick the answer that is correct to within an order-of-magnitude (in other words, pick the closest power of ten).

Question 5 (15 points)

An Jupiter-sized planet transits in front of an A0 main-sequence star. By how much will the star dim when the planet passes in front of it?

a) about 0.001%
b) about 0.01%
c) about 0.1%
d) about 1%
e) about 10%

The amount that a star dims when a planet passes in front of it is just given by the ratio of the planet to stellar radius squared, i.e.,

$$\text{Dimming} = \left(\frac{\text{Radius of Planet}}{\text{Radius of Star}}\right)^2 .$$

The radius of an A0 star, according to my notes, is 2.3 times the radius of the Sun. Therefore,

$$\text{Dimming} = \left(\frac{\text{Radius of Jupiter}}{2.3 \times \text{Radius of Sun}}\right)^2 = \left(\frac{71492\ \text{km}}{2.3 \times 695500\text{km}}\right)^2 = 1.997 \times 10^{-3} \sim 10^{-3}$$

Answer c) about 0.1%

Question 6 (15 points)

An Mars-sized planet transits in front of an G2 main-sequence star. By how much will the star dim when the planet passes in front of it?

a) about 0.001%
b) about 0.01%
c) about 0.1%
d) about 1%
e) about 10%

The radius of an G2 star is just the radius of the Sun (since the Sun is a G2 star). Therefore,

$$\text{Dimming} = \left(\frac{\text{Radius of Mars}}{\text{Radius of Sun}}\right)^2 = \left(\frac{3397\ \text{km}}{695500\text{km}}\right)^2 = 2.386 \times 10^{-5} \sim 10^{-5} .$$

Answer a) about 0.001%

Question 7 (15 points)

An Mars-sized planet transits in front of an M8 main-sequence star. By how much will the star dim when the planet passes in front of it?

a) about 0.001%
b) about 0.01%
c) about 0.1%
d) about 1%
e) about 10%

The radius of an M8 star, according to my notes, is 0.15 times the radius of the Sun. Therefore,

$$\text{Dimming} = \left(\frac{\text{Radius of Mars}}{0.15\times\text{Radius of Sun}}\right)^2 = \left(\frac{3397\text{ km}}{0.15\times 695500\text{km}}\right)^2 = 1.060\times10^{-3} \sim 10^{-3}$$

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Answer c) about 0.1%

Extra Credit Questions:

Question 8 (30 points)

Assume that the tidal locking radius as a function of the mass of the primary star is given by,

$$d_{lock} = 0.4\,\text{AU}\left(\frac{M}{M_{Sun}}\right)^{1/2},$$

where M is the mass of the star. For what primary masses will any planets in the optimistic habitable zone be tidally locked? (*Hint: Adopt the mass-luminosity relationship I showed in class to derive a relationship between the habitable zone distance and primary mass.)*

a) 0.11 M_{Sun} and below
b) 0.25 M_{Sun} and below
c) 0.38 M_{Sun} and below
d) 0.89 M_{Sun} and below

For the Sun, the optimistic habitable zone extends out to 1.7 AU. We can derive how this scales with the mass of the star using the fact that, at fixed equilibrium temperature, the distance should scale with the star's luminosity as,

$$\frac{d_{outer}}{d_{outer,Sun}} = \left(\frac{L_{Star}}{L_{Sun}}\right)^{1/2},$$

and adopting the mass-luminosity relation from class,

$$\frac{L_{Star}}{L_{Sun}} = \left(\frac{M}{M_{Sun}}\right)^4,$$

We can write

$$\frac{d_{outer}}{d_{outer,Sun}} = \left(\frac{L_{Star}}{L_{Sun}}\right)^{1/2} = \left(\left(\frac{M}{M_{Sun}}\right)^{4}\right)^{1/2} = \left(\frac{M}{M_{Sun}}\right)^{2}.$$

To find the largest stellar mass for which all planets in the habitable zone are tidally locked, we can just equate d_{outer} in the expression above with d_{lock} in the expression for the tidal locking radius given in the question text. For the left hand side of the equation above, we get,

$$\frac{d_{outer}}{d_{outer,Sun}} = \frac{d_{lock}}{d_{outer,Sun}} = \frac{0.4\,\mathrm{AU}}{1.7\ \mathrm{AU}}\left(\frac{M}{M_{Sun}}\right)^{1/2} = 0.2353\left(\frac{M}{M_{Sun}}\right)^{1/2},$$

which we can now equate to the right hand side of the equation,

$$0.2353\left(\frac{M}{M_{Sun}}\right)^{1/2} = \left(\frac{M}{M_{Sun}}\right)^{2}.$$

All that remains is to solve for M. After a small amount of algebra, we get

$$\left(\frac{M}{M_{Sun}}\right)^{3/2} = 0.2353,$$

which we can easily solve for M,

$$M = (0.2353)^{2/3} M_{Sun} = 0.38 M_{Sun}.$$

Answer c) 0.38 M_{Sun} and below

Question 9 (30 points)

The solar-like star Alpha Centauri is one of the closest stars to the Sun. If Alpha Centauri were orbited by an Earth-mass planet near the outer edge of the optimistic habitable zone, by what angle would Alpha Centauri appear to move back-and-forth as the Earth-mass planet completes its orbit?

a) about 1 arcsecond
b) about 0.1 arcseconds
c) about 10^{-5} arcseconds
d) about 0.01 arcseconds
e) about 10 arcseconds

As the Earth-mass planet orbits Alpha Centauri, the star will orbit about the common center-of-mass of the star-planet system. The center of mass is offset a distance from the center of the star given by,

$$d_{CoM} = a\frac{M_{Planet}}{M_{Star}},$$

Where a is the semimajor axis of the planet. We can assume that Alpha Centauri and the Sun are identical (this is not quite true, but close enough for our purposes). Then the outer edge of the optimistic habitable zone for Alpha Centauri is the same as that for the Sun, namely 1.7 AU. The semimajor axis of the planet's orbit is then 1.7AU. Since the center-of-mass distance is equal to the radius of the orbit, the star will move by twice this value, namely,

$$d_{wobble} \sim 2d_{CoM} = 2a\frac{M_{Planet}}{M_{Star}} \sim 2\times 1.7\,\mathrm{AU}\frac{5.9742\times 10^{24}\,\mathrm{kg}}{1.9884\times 10^{30}\,\mathrm{kg}} \sim 10^{-5}\,\mathrm{AU}.$$

Where we've assumed the mass Alpha Centauri is the same as that of the Sun. Alpha Centauri is located at the distance of about 1.34 parsecs. At this distance, the wobble of the star corresponds to an angle θ on the sky of,

$$\theta = \frac{d_{wobble}}{1.34\ \mathrm{parsecs}} = \frac{10^{-5}\,\mathrm{AU}}{1.34\ \mathrm{parsecs}} \approx 10^{-5}\,\mathrm{arcseconds}.$$

Recall our first homework assignment, and that one parsec is defined to be the distance at which one AU subtends on arcsecond.

Answer c) about 10^{-5} arcseconds