Taqseer Khan

Abstract

Operator theory occupies an important place in Analysis because of its significant applications in various fields of Mathematics as well in other natural sciences. An operator $T : X \to Y$ is called compact if the image of each bounded subset of X is precompact in Y. Compactness of an operator is a powerful notion but it occurs not very often. Several other notions which are near to this concept have been studied. One of them is strict singularity. An operator T is said to be strictly singular if its restriction to any infinite dimensional subspace does not induce an isomorphism. The notion is widely studied, for intance in [1, 2, 3, 4]. It is well known that compactness $\implies$ finite strict singularity $\implies$ strict singularity. But the converse is false. We study the following Volterra operator $V : L^1[0,1] \to C[0,1]$ defined by

$$V(f)(x) = \int_0^x f(t)dt.$$

To study the asymptotic behaviour of this operator we associate a sequence of numbers, called s- numbers. There are several s-numbers, for example, approximation numbers $a_n(V)$, isomorphism numbers $i_n(V)$, Gelfand numbers $c_n(V)$, Bernstein numbers $b_n(V)$, Mityagin numbers $m_n(V)$, Kolmogorov numbers $d_n(V)$ etc.. In general, it is known that $a_n(v) \geq d_n(v) \geq m_n(v) \geq i_n(V)$ We are investigating for the bounds of the Mityagin numbers and trying to establish the equality

$$b_(V) = m_n(V) = i_n(V).$$

References

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