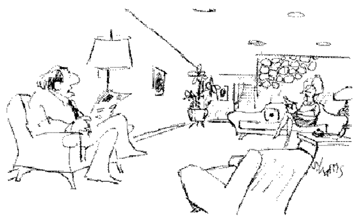
The Density of the Universe



"The whole universe is expanding, so why be surprised that we're drifting apart?"

Wednesday, November 18

Hand in Problem Set 6; Pick up Problem Set 7

Because space is nearly flat (Euclidean) today, we know the average density is close to the critical density.

$$\rho_{\text{crit}} = \frac{3\,\text{H}_0^2}{8\pi\,\text{G}} = 10^{-26}\ \text{kg/m}^3$$

The density can be provided by either mass or energy:

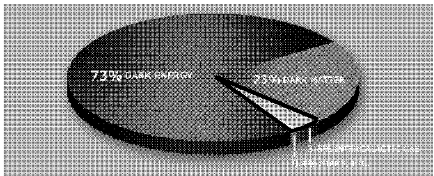
mass density:

$$\rho_{\text{crit}} = 10^{-26}\ \text{kg/m}^3$$

energy density:

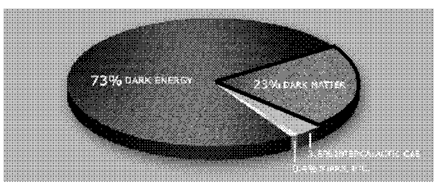
$$\rho_{\text{crit}}\text{c}^2 = 9\times10^{-10}\ \text{joules/m}^3$$

Right now, only **4%** of the density is provided by **ordinary matter** (protons, neutrons, electrons).



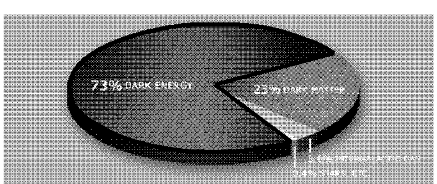
Some ordinary matter is in stars, but most is in low-density intergalactic gas.

Right now, **23%** of the density is provided by **dark matter** (WIMPs, neutrinos).



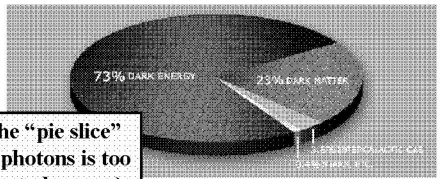
WIMPs, neutrinos, protons, neutrons, & electrons are particles with mass.

Right now, **73%** of the density is provided by **dark energy**.



Dark energy is an energy field, and is not made of massive particles.

Right now, **0.005%** of the density is provided by **photons**.



(The "pie slice" for photons is too thin to be seen.)

Photons are particles with **energy**, but not mass.

The futility of stars.

Stars have been converting H to He for 13 billion years. However, most helium was created at t ≈ 3 minutes.

Stars have been making photons for 13 billion years. However, most light is left over from t ≈ 400,000 years.

The total density of the universe was greater in the past than it is now.

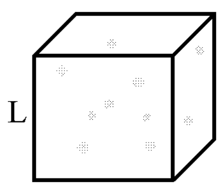
hot dense	→	cold tenuous

The density of different components (photons, matter, dark energy) varied at different rates as space expanded.

Dark energy remains **constant** in density as the universe expands.

Current density of dark energy =
$0.73\ \rho_{crit}\ c^2$
$= 0.73\ (9 \times 10^{-10}$ joules/m$^3)$
$= 6.6 \times 10^{-10}$ joules/m^3

How does density of **matter** evolve with time?



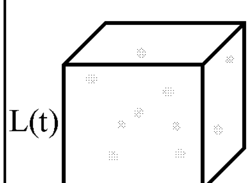
Consider a cube with sides of length L.

Volume of cube = L^3

Mass of particles in cube = M

Density of particles in cube (ρ)= M/L^3

Let's suppose the cube is expanding along with the universe.

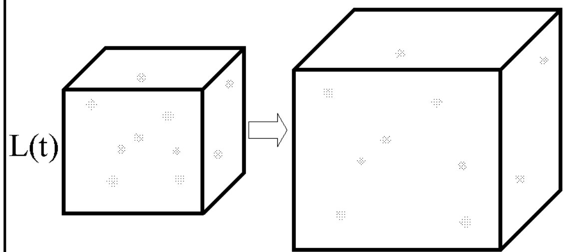


Length of side of cube
$= L(t) = L_0 \times a(t)$

L_0 = Current length of side

$a(t)$ = scale factor of universe

As the cube expands, the number of particles is constant. The mass per particle is constant.



Thus, the total mass M within the cube is **constant**.

The density of matter (ρ) in an expanding universe:

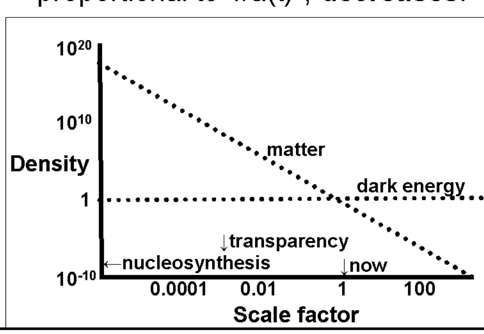
$$\rho(t) = \frac{M}{L(t)^3} = \frac{M}{L_0^3 a(t)^3} = \frac{\rho_0}{a(t)^3}$$

ρ_0 = Current density (0.27×10^{-26} kg/m^3)

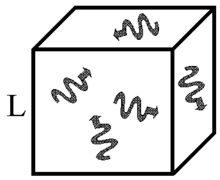
a(t) = scale factor of universe

$\rho(t)$ = Density at time t

As the universe expands, a(t) increases. Thus, the density of matter, proportional to $1/a(t)^3$, **decreases**.



How does energy density of **photons** evolve with time?



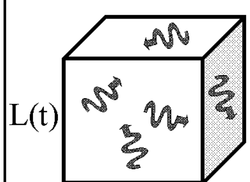
Consider a cube with volume L^3.

Number of photons in cube = N

Energy per photon = E

Energy density of photons in cube (ρc^2)= $N \times E/L^3$

Let's suppose the cube is expanding along with the universe.

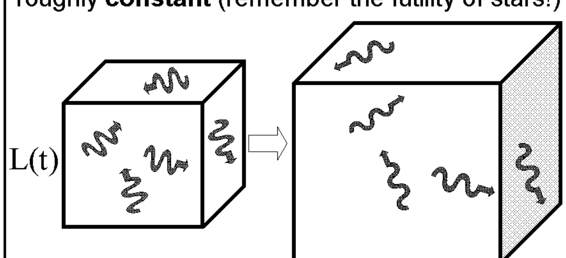


Length of side of cube = $L(t) = L_0 \times a(t)$

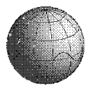
L_0 = Current length of side

$a(t)$ = scale factor of universe

As the cube expands, the number of photons is roughly **constant** (remember the futility of stars!)



However, the energy E of each photon is **not** constant.



As space expands, the wavelength of light expands. Longer wavelength → lower frequency → smaller photon energy.

Wavelength: $\lambda(t) = \lambda_0 \times a(t)$

Frequency: $f(t) = f_0 / a(t)$

Photon energy: $E(t) = E_0 / a(t)$

The energy density of photons (ρc^2) in an expanding universe:

$$\rho(t)c^2 = \frac{N\,E(t)}{L(t)^3} = \frac{N\,E_0 / a(t)}{L_0^3 a(t)^3} = \frac{\rho_0 c^2}{a(t)^4}$$

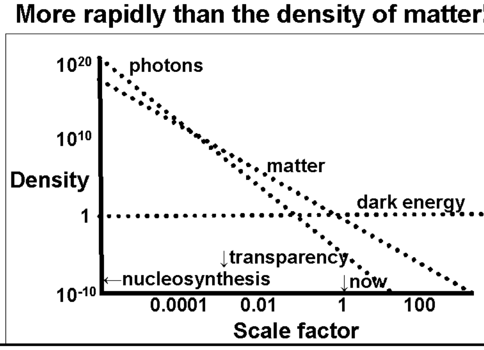
$\rho_0 c^2$ = Current density (0.00005 $\rho_{crit}c^2$)

$a(t)$ = scale factor of universe

$\rho(t)c^2$ = Density at time t

As the universe expands, energy density of photons decreases as $1/a(t)^4$.

More rapidly than the density of matter!



When the scale factor was **a** < **0.0003**, & age of the universe was **t** < **70,000 years**, the universe was **radiation-dominated**.

"Radiation-dominated" simply means that photons provided most of the density.

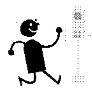
When **0.0003** < **a** < **0.7**, & **70,000 years** < **t** < **10 billion years**, the universe was **matter-dominated**.

"Matter-dominated" means that ordinary & dark matter provided most of the density.

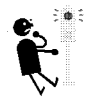
Now that **a** > **0.7** & **t** > **10 billion years**, the universe is **dark-energy-dominated**.

Photons & matter have finally been diluted to the point where dark energy provides most of the density.

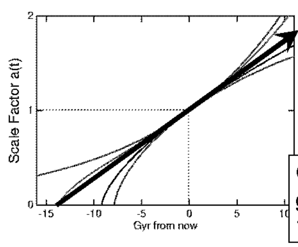
We have just reached the stage where expansion is **speeding up** (under the influence of dark energy).



At earlier times, gravity acting on photons & matter caused the expansion to **slow down**.



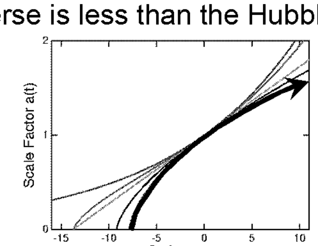
In an empty universe ($\rho=0$), the age of the universe is exactly equal to the Hubble time.



Gyr = gigayear = 1 billion years

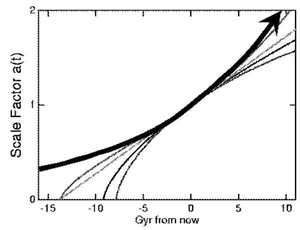
Expansion of this universe is coasting – relative speed of any 2 points is constant.

In a flat universe containing only photons & matter, the age of the universe is less than the Hubble time.



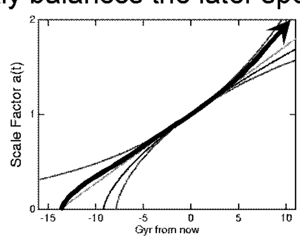
Expansion is slowing down – relative speed of any 2 points was faster in the past.

In a flat universe containing lots of dark energy, the age of the universe is greater than the Hubble time.



Expansion is speeding up – relative speed of any 2 points was slower in the past.

What about the real universe? Amusingly, the early slow-down almost exactly balances the later speed-up.



Hubble time = 14 billion years.
Age of universe = 13.7 billion years.

Friday's Lecture:

Destiny of the Universe

Reading:

Chapters 10 & 11