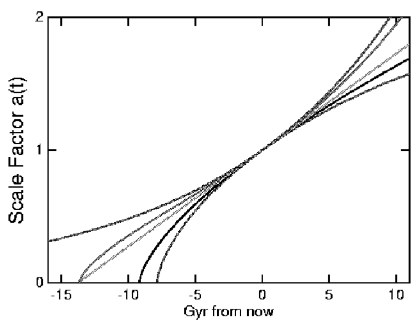
The Destiny of the Universe

Friday, November 20

The universe is expanding: that is, the scale factor a(t) is **increasing** with time.

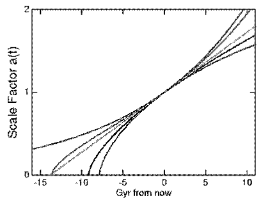


Naïve question:
Why is the universe expanding?

Naïve answer:
The universe is expanding today because it was expanding yesterday.

(Objects in motion tend to remain in motion at constant velocity.)

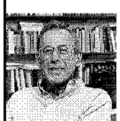
The universe was expanding yesterday because it was expanding the day before yesterday.



…And so forth, back to the Big Bang (the beginning of expansion).

Naïve question:
Why did the universe **start** expanding?
(What put the "bang" in the Big Bang?)

Naïve answer:
We don't know.



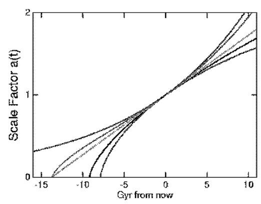
"The big bang theory describes how our universe is evolving, not how it began."
– Jim Peebles

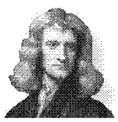
The origin of the expansion
(in Newton's terms, the force that caused the initial acceleration)
was in the very early universe.

To describe the very early universe, we need a good theory of "quantum gravity".

We haven't got one.

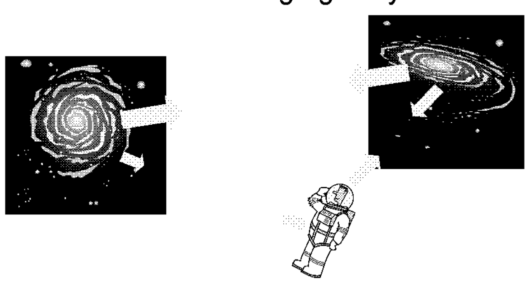
Another naïve question:
Will the universe expand forever?



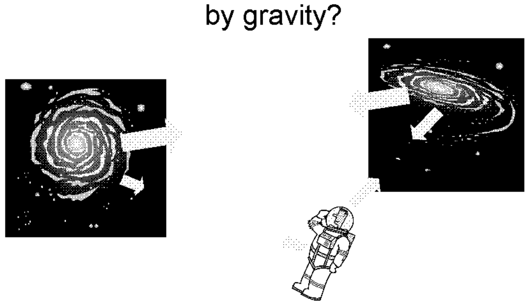


(A force can make motion speed up or slow down.)

What would Newton say? The universe is full of massive objects attracting each other through gravity.

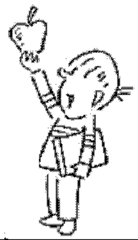


Gravitational attraction slows the expansion. Can the expansion ever be brought to a halt by gravity?



Start with a related Newtonian problem:

A boy standing on the Earth throws an apple upward: initially, the distance between apple & Earth is **increasing**.



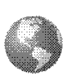
Is the attractive force between apple & Earth enough to stop the apple from rising?

What goes up must come down.

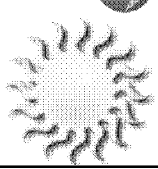
…unless it's traveling faster than the **escape velocity**.

Escape velocity from a planet (or star) depends on its **mass (M)** & **radius (r)**.

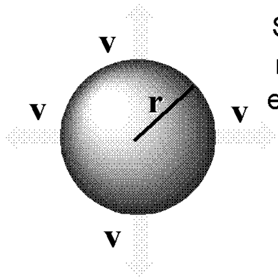
$$v_{esc} = \sqrt{\frac{2GM}{r}}$$



Escape velocity from **Earth**:
11 km/sec = 25,000 mph

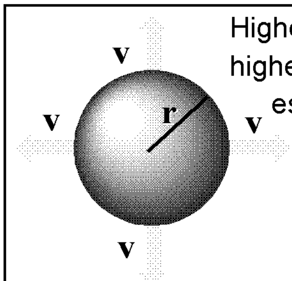


Escape velocity from **Sun**:
600 km/sec = 1,400,000 mph



Suppose a sphere of matter (radius = r) is expanding outward at a speed v.

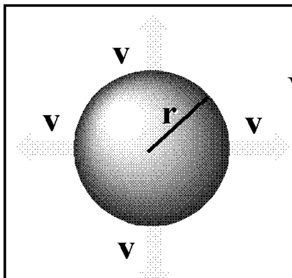
If expansion speed is greater than escape speed ($v > v_{esc}$), the sphere will expand forever.



Higher density ρ leads to a higher mass M, & a higher escape velocity v_{esc}.

$$M = \frac{4\pi}{3} r^3 \rho$$

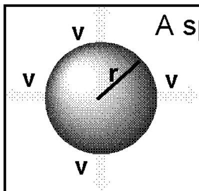
$$v_{esc} = \sqrt{\frac{2GM}{r}}$$



$$v_{esc} = \sqrt{\frac{2G}{r}\frac{4\pi}{3} r^3 \rho}$$

$$v_{esc} = \sqrt{\frac{8\pi}{3} G r^2 \rho}$$

Large dense spheres have a high escape velocity.



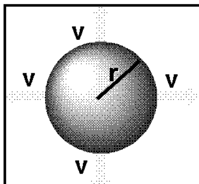
A sphere is expanding at exactly its escape velocity when

$$v = \sqrt{\frac{8\pi}{3} G r^2 \rho}$$

That is, when its density is

$$\rho = \frac{3}{8\pi G} \frac{v^2}{r^2}$$

This is the critical density for a sphere of radius r expanding at speed v.



$$\rho_{crit} = \frac{3}{8\pi G} \frac{v^2}{r^2}$$

Suppose our sphere of matter is part of the expanding universe, so that $v = H_0 r$.

$$\rho_{crit} = \frac{3}{8\pi G} \frac{(H_0 r)^2}{r^2}$$

$$\rho_{crit} = \frac{3 H_0{}^2}{8\pi G}$$

We found this result using old-fashioned **Newtonian** physics.

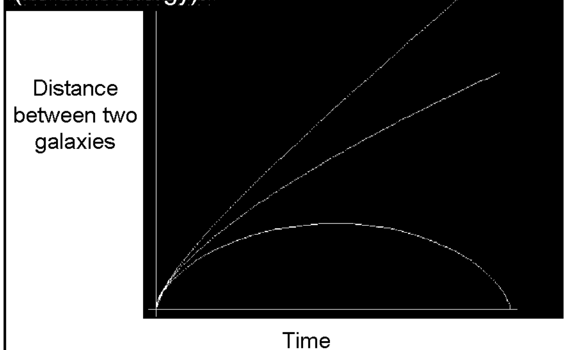
However, it’s the same as the critical density required to make a flat universe, according to Einstein!!

Newton says:
Destiny of the universe depends on the ratio of its **density** to the **critical density**.

$$\frac{\rho}{\rho_{\text{crit}}} = \Omega$$

Omega (Ω) is also called the "**density parameter**".

Newtonian result (no dark energy).

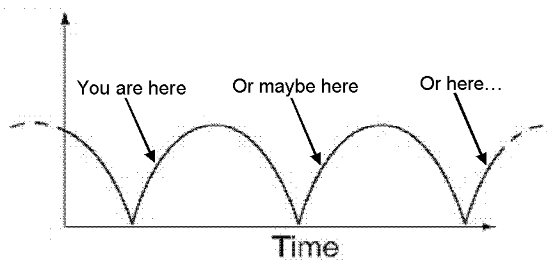


Newtonian result (no dark energy).

Ω>1 (density greater than critical):
The Big Crunch
(recollapse, becoming hotter)

Ω≤1 (density less than or equal to critical):
The Big Chill
(perpetual expansion, becoming cooler)

Amusing speculation: perhaps a Big Crunch would lead to a Big Bounce.



Einstein says:
Curvature of the universe depends on the ratio of its **density** to the **critical density**.

$$\Omega = \frac{\rho}{\rho_{crit}}$$

Now the density ρ includes dark energy and photons as well as matter.

Relativistic result (with dark energy).

$\Omega>1$ (density greater than critical):
Positive curvature

$\Omega<1$ (density less than critical):
Negative curvature

$\Omega=1$ (density equal to critical):
Flat

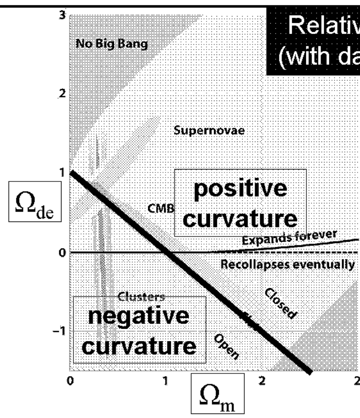
Einstein says:

Destiny of the universe depends on the amounts of matter & dark energy today.

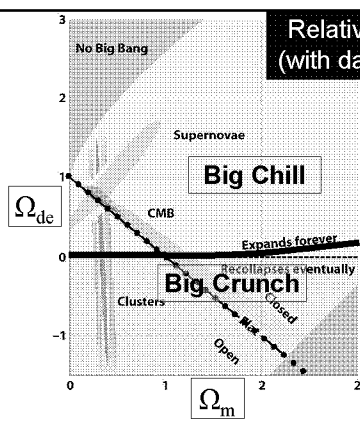
$$\Omega_m = \frac{\rho_{matter}}{\rho_{crit}} \qquad \Omega_{de} = \frac{\rho_{dark\ energy}}{\rho_{crit}}$$

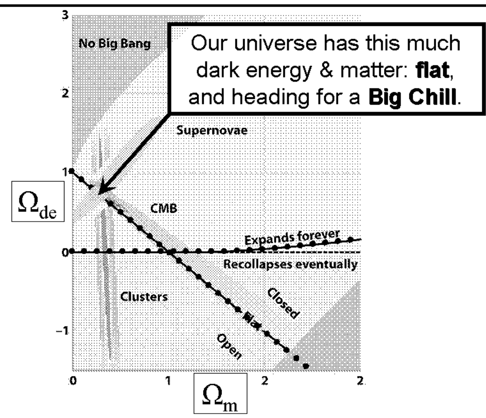
Today, $\Omega_m = 0.27$, $\Omega_{de} = 0.73$

Relativistic result (with dark energy).



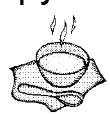
Relativistic result (with dark energy).





Monday's Lecture:

Why is the Universe Lumpy?



Reminders:

Pick up all your corrected problem sets!
Have you read chapters 1 – 12 ?
Problem Set 7 is due on Wednesday.