1) You are given a ticket for running a red traffic light. For an observer halted at the red light, the light emits a wavelength $\lambda_0 = 700$ nm. You tell the traffic cop that because you were approaching the light, the Doppler shift made it appear green ($\lambda = 500$ nm). How fast would you have been going if this smart-aleck explanation had been true? Please express this speed in units of miles per hour. [Hint: There are 1.6093 kilometers in a mile.]
First,

$$\Delta\lambda = \lambda - \lambda_0 = 500 \text{ nm} - 700 \text{ nm} = -200 \text{ nm}.$$

In words, the wavelength decreased by 200 nm, making it appear green (we would call this a blueshift). $\Delta\lambda < 0$ implies that we're **moving toward** the traffic light (as expected). This is a useful consistency check. By convention, if the relative velocity of two objects is negative (i.e., $v < 0$), then they are getting closer together. This is because the distance that separates them is *decreasing* as time goes on.

Next we determine the driver's velocity (v). The wavelength change is related to velocity by the following formula:

$$\frac{\Delta\lambda}{\lambda_0} = \frac{v}{c},$$

where $\Delta\lambda$ is the change in wavelength, λ_0 is the wavelength at rest (in the lab), c is the speed of light, and v is radial velocity (along the line of sight). Multiplying both sides of the equation by c, we find:

$$v = c\,\frac{\Delta\lambda}{\lambda_0}\,.$$

Plugging in the numbers from above and $c = 3.0 \times 10^5$ km/s, we have:

$$v = (3.0\times10^5 \text{ km/s}) \left(\frac{-200 \text{ nm}}{700 \text{ nm}}\right) = (3.0\times10^5 \text{ km/s})\times\left(\frac{-2}{7}\right) = -8.571\times10^4 \text{ km/s}$$

Note that the units of wavelength (nm) canceled out. Lastly, we must convert km/s to miles per hour (mph). The hint indicates that 1 mi = 1.6093 km and we know that 1 hr = 3600 s (1 hr = 60 min $\times$ 60 s/min = 3600 s). Combining these two we get:

$$1 \text{ km/s} = 1 \text{ km/s} \times \left(\frac{1 \text{ mi}}{1.6093 \text{ km}}\right) \times \left(\frac{3600 \text{ s}}{1\text{hr}}\right) \approx 2237 \text{ mph}$$

Putting all this together we find:

$$v = \left(-8.571 \times 10^4 \text{ km/s}\right) \times \left(\frac{2237 \text{ mph}}{1 \text{ km/s}}\right) \approx -1.92 \times 10^8 \text{ mph}$$

2) Astronomers doing a "star census" of the solar neighborhood have found that there are 340 stars within a distance $r = 10\,\mathrm{pc}$ of the Earth. Compute the volume of a sphere of radius $r = 10\,\mathrm{pc}$ centered on the Earth. Compute the average number of stars per cubic parsec within this sphere.

The first part is quite straightforward. From geometry, recall that the formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$, where $\pi \approx 3.14$. Plugging $r = 10\,\mathrm{pc}$ into this gives us:

$$V = \frac{4}{3}\,\pi\,r^3 = \frac{4}{3}\,\pi\,(10\,\mathrm{pc})^3 = \frac{4}{3}\,\pi\,(1000\,\mathrm{pc}^3) = \frac{4000\,\pi}{3}\,\mathrm{pc}^3 \approx 4189\,\mathrm{pc}^3$$

Next, we have to compute the average number of stars within this volume. This average is simply the total number of stars inside the volume (340, from above) divided by the total volume. Putting these together we have:

$$n = \frac{\textit{total stars}}{\textit{total volume}} = \frac{340}{4189\,\mathrm{pc}^3} \approx 0.081\ \mathrm{stars/pc}^3 = 0.081\,\mathrm{pc}^{-3}$$

Rounding, significant figures, and all that:

In any quantitative field, it's important to round your answers to an appropriate number of decimal places (a.k.a., "significant figures"). The motivation here is that the number of decimal places you provide when reporting some value should (roughly) indicate the accuracy of that value. While this is not absolutely crucial, it's good practice and definitely worth understanding.

For example, suppose someone hands you a ruler and asks you to measure the long side of a sheet of standard 8.5x11" paper. You would probably come up with a number very close to 11". Maybe 10.9", maybe 11.1". You would NOT (I hope), however, give an answer to the tune of 11.0823475".

In the same fashion, you should try to round off your answers to (at least approximately) match the problem. In practice, this is fairly simple. A good rule of thumb is: examine all the numbers that entered your calculation, find the one with the least number of specified decimal places (probably the least accurate), and round your answer to about that same number of digits. Here, the "number of digits" is how many you would keep in scientific notation. As a result, 35, 3.5, and 0.035 all have 2 digits, since they would be written as 3.5×10^N.

One last point: whenever possible, you should only round your final answer. In practice, this means leave all the digits in your calculator but only copying the first few onto paper. For example, in the question above, I've kept 4 digits in the intermediate result V. For the final n, however, I have only reported 2 digits ($8.1 \times 10^{-2}\,/\,\mathrm{pc}^3$), a compromise between the 1 digit of $r = 10$ and the two digits of $N = 340$ stars. In practice, the exact number of digits is not important. Understanding that there is a difference and choosing a reasonable number is just fine.

3) As shown in the textbook (Figure 3.25), the Milky Way Galaxy can be approximated as a cylinder whose diameter is $d = 100,000\,\mathrm{ly}$ and whose thickness is $h = 2000\,\mathrm{ly}$. What are the diameter and thickness in units of parsecs? What is the volume of the Milky Way Galaxy in units of cubic parsecs? Assume that the average number of stars per cubic parsec in the Milky Way Galaxy is the same as the number of stars per cubic parsec in the solar neighborhood: what, in this case, is the total number of stars in the Milky Way Galaxy?

The first step is conversion of the galaxy dimensions from lightyears (ly) to parsecs (pc). To do this, recall that $1\,\mathrm{pc} = 3.26\,\mathrm{ly}$. Using this, we convert units as follows:

$$d = 100,000\,\mathrm{ly} \times (\,1\,\mathrm{pc}\,/\,3.26\,\mathrm{ly}) = 30,675\,\mathrm{pc} \approx 30,700\,\mathrm{pc}$$

$$h = 2,000\,\mathrm{ly} \times (\,1\,\mathrm{pc}\,/\,3.26\,\mathrm{ly}) = 613.5\,\mathrm{pc} \approx 613\,\mathrm{pc}$$

Next we compute the volume of the galaxy. The volume of a cylinder can be calculated with $V = \pi\, r^2\, h$, where r is the cylinder radius. Please note that since $\pi\, r^2$ is the area of a circle, this formula is effectively just $base \times height$). If you want a formula that involves the cylinder diameter, just plug in $d = 2r$ (so $r = d/2$) to find:

$$V = \pi\, r^2\, h = \pi\, h \left(\frac{d}{2}\right)^2 = \frac{\pi}{4}\, h\, d^2.$$

To find volume, plug in the diameter (d) and height (h) from above:

$$V = \frac{\pi}{4}\, h\, d^2 = \frac{\pi}{4} \times (613.5\,\mathrm{pc}) \times (30,675\,\mathrm{pc})^2 \approx 4.53 \times 10^{11}\,\mathrm{pc}^3$$

Alternatively (possibly a shortcut), you could do the volume calculation in lightyears using the nice round numbers and then convert your answer appropriately. I did this using $r = d/2 = 50,000\,\mathrm{ly} = 5 \cdot 10^4\,\mathrm{ly}$ and $h = 6,000\,\mathrm{ly} = 6 \cdot 10^3\,\mathrm{ly}$. Plugging these values in in, I get:

$$V = \pi\, r^2\, h = \pi \cdot (5 \cdot 10^4\,\mathrm{ly})^2 \cdot (2 \cdot 10^3\,\mathrm{ly}) = \pi \cdot (2 \cdot 10^3) \cdot (25 \cdot 10^8)\,\mathrm{ly}^3 = 50\pi \cdot 10^{11}\,\mathrm{ly}^3$$

You could then convert this using $1\,pc = 3.26\,\mathrm{ly}$ (don't forget to cube it) as follows ($3.26^3 \approx 34.65$):

$$V = 50\pi \cdot 10^{11}\,\mathrm{ly}^3 \times \left(\frac{1\,\mathrm{pc}}{3.26\,\mathrm{ly}}\right)^3 = \frac{50\pi}{34.65} \cdot 10^{11}\,\mathrm{pc}^3 = 4.53 \times 10^{11}\,\mathrm{pc}^3$$

Lastly, the number of stars is found with $N_{stars} = n \times volume$, where n is the number of stars / pc^3. We calculated this in question 2 and found that $n = 0.081$ stars/pc^3. You could also use $n = 0.1\,\mathrm{pc}^{-3}$ from the class notes. Plugging in, find:

$$N_{stars} = n \times \mathrm{Vol} = (\,0.81\,\mathrm{stars/pc}^3\,) \cdot (4.53 \cdot 10^{11}\,\mathrm{pc}^3) = 3.67 \cdot 10^{10}\,\mathrm{stars\ for}\ n = 0.081\,\mathrm{pc}^{-3}$$

$$N_{stars} = n \times \mathrm{Vol} = (\,0.1\,\mathrm{stars/pc}^3\,) \cdot (4.53 \cdot 10^{11}\,\mathrm{pc}^3) = 4.53 \cdot 10^{10}\,\mathrm{stars\ for}\ n = 0.1\,\mathrm{pc}^{-3}$$

4) Suppose you have a friend, "Flat-Earth Fred," who believes that the Earth is flat rather than spherical. (He believes that all the NASA photographs showing a spherical Earth are faked; he wants evidence that he can see directly with his own eyes.)

First, describe what evidence you could provide, without having to leave Columbus, that the Earth is spherical. [Hint: On June 29, 2010, there will be a lunar eclipse visible from Columbus.]

Second, suppose that you have a private jet and can fly anywhere in the world where there's an airport. With this added mobility, what additional evidence could you provide to convince Fred that the Earth is spherical?

From Columbus, you would show Fred the lunar eclipse. Your response should more or less include all the following pieces:
– Lunar eclipses help demonstrate that the Earth is spherical.
– Lunar eclipses occur when the (full) Moon passes through the Earth's shadow. By looking, we deduce that the Earth's shadow is roughly *circular* in shape. Importantly, during lunar eclipses, Earth's shadow always appears circular.
– The only object that always casts a circular shadow is a sphere.

Separately, you might reason with him about the effects of gravity, pointing out that the sphere is the preferred shape to minimize energy. Although this isn't something Fred can see directly, it might bolster your argument.

With a private jet at your disposal, your options are wide open. In a jet, you could:
– Keep flying in a straight line (provided enough fuel) and arrive back where you started. You would point out to Fred that you did this without passing any "corners" along the way.
– With a decent camera and enough altitude, you could possibly observe the Earth's curvature, though this is apparently quite difficult.

A variety of other options exist (not all of which are covered here). You could select a destination with a beach so that you can show that ships disappear hull-first over the horizon. This cannot be observed unless the surface of the planet is curved.

With the right measurements (e.g., diffraction of starlight or Doppler velocity), you could possibly measure the rotation speed of the Earth and demonstrate that it decreases as you get farther from the Equator. Assuming that the Earth is a solid body, this would quickly lead you to the conclusion that we inhabit a sphere.