1) Potassium-40 is an unstable atomic nucleus; it decays to Argon-40 with a half-life of 1.3 billion years. Suppose that a rock contains 1,000 Potassium-40 atoms at the time it forms. How many Potassium-40 atoms will be left after 1.3 billion years? How many will be left after 2.6 billion years? How many will be left after 3.9 billion years?

By definition, exactly half of the nuclei will decay (leaving half behind) with each half-life (λ) that passes. Denoting the initial number of particles as N_0, we would find that the number of remaining particles is

$$\text{Number of particles remaining } = N(t) = N_0 \cdot \left(\frac{1}{2}\right)^{t/\lambda} = N_0 \cdot \left(\frac{1}{2}\right)^{n_\lambda},$$

where $n_\lambda = t/\lambda$ is shorthand for the number of half-lives that have elapsed since the clock started.

Using the shorthand formula above, we find that:

- After 1.3 Gyr (1 half-life), have 500 atoms left. $\leftrightarrow$ $1000 \cdot \left(\frac{1}{2}\right)^1 = \frac{1000}{2} = 500.$
- After 2.6 Gyr (2 half-lives), have 250 atoms left. $\leftrightarrow$ $1000 \cdot \left(\frac{1}{2}\right)^2 = \frac{1000}{4} = 250.$
- After 3.9 Gyr (3 half-lives), have 125 atoms left. $\leftrightarrow$ $1000 \cdot \left(\frac{1}{2}\right)^3 = \frac{1000}{8} = 125.$

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2) The "life span" of the Sun is 10 billion years; that is, at the time it formed, it contained enough hydrogen to power nuclear fusion for 10 billion years. In general, the amount of hydrogen fuel that a star contains is proportional to its mass M; the rate at which it consumes that fuel is proportional to its luminosity L. This implies that the life span t of the star is proportional to M/L.

The star Altair, like the Sun, is powered by the fusion of hydrogen to helium. The mass of Altair is $M_{Altair} = 1.7 M_{sun}$. The luminosity of Altair is $L_{Altair} = 10.7 L_{sun}$. Is the life span of Altair *shorter* or *longer* than that of the Sun? What is the approximate life span of Altair, in billions of years?

For starters, we know that $t \propto M/L$ for stars ($\propto$ means "proportional to") and that $t_{sun} = 10$ billion years. Keep in mind that $t \propto M/L$ is NOT the same as $t = M/L$! In general, dividing units of mass (M) by units of luminosity (L) will NOT give you time units for t ! Proportionality relationships like the one above typically need a constant (which might or might not include units) in order to become an equality.

However, since we already know the lifetime of the Sun from the question, we can quickly find the new answer by comparison. From the proportionality $t \propto M/L$, we can say (taking ratios):

$$(\, t_{altair} \,/\, t_{sun} \,) = \frac{M_{altair} \,/\, M_{sun}}{L_{altair} \,/\, L_{sun}}$$

From the question, we know that $M_{Altair}/M_{sun} = 1.7$ and $L_{altair}/L_{sun} = 10.7$. Plugging these values in, we find:

$$t_{altair} \,/\, t_{sun} = \frac{1.7}{10.7} = 0.159 \quad \rightarrow \quad t_{altair} \approx 0.16 \cdot t_{sun}.$$

From the above values, it is clear that $t_{altair} < t_{sun}$, meaning the lifespan of Altair is shorter than the lifespan of the Sun. Plugging the Sun's lifetime in to the above relation gives Altair's lifetime:

$$t_{altair} = 0.16 \cdot t_{sun} = 0.16 \cdot 10 \text{ billion years } = \boxed{\text{1.6 billion years}}$$

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3) There are 411,000,000 cosmic microwave photons per cubic meter of the universe. The average energy of a cosmic microwave photon is very small: only $E = 1.02 \times 10^{-22}$ joules. What is the energy density of the Cosmic Microwave Background, in joules per cubic meter? Using Einstein's relation, $E = mc^2$, what is the equivalent *mass* density, in kilograms per cubic meter? What fraction of the critical density, $\rho_{crit} = 10^{-26}$ kg/m^3, does this density represent?

First, compute the energy density of the CMB. We know the number of photons per unit volume ($n_{CMB} = 4.11 \times 10^8$ photons / m^3) and the average energy that each carries ($E = 1.02 \times 10^{-22}$ J / photon). Multiplying these together (note that the units work out) gives us the CMB energy density:

$$\begin{aligned}
\text{CMB energy density} &= n_{CMB} \cdot E \\
\text{CMB energy density} &= (4.11 \times 10^8\ \cancel{\text{photons}}\ /\text{m}^3) \cdot (1.02 \times 10^{-22}\ \text{J} / \cancel{\text{photon}}) \\
&\boxed{\text{CMB energy density} = 4.19 \times 10^{-14}\ \text{J} / \text{m}^3}
\end{aligned}$$

Next we'll calculate the equivalent mass density ρ_m using $E = mc^2$. To simplify this calculation, recall that:

$$c^2 = (3.0 \times 10^8\ \text{m} / \text{s})^2 = 9.0 \times 10^{16}\ \text{J} / \text{kg}$$

CAUTIONARY NOTE: Choosing the correct conversion factor is **extremely important**. When using the relation $E = mc^2$, take care with your units!!! Since 1 Joule = 1 kg m^2 / s^2, you **MUST** use $c = 3.0 \times 10^8$ m/s !! If you have E in Joules, m in kg, but use $c = 3.0 \times 10^5$ km/s, the units DO NOT WORK as expected, because km/s does not properly cancel the m/s built into Joules! This is a very common mistake.

Converting the previously-found CMB energy density with the c^2 value above, find:

$$\text{CMB mass density} = \rho_m = \frac{\text{CMB energy density}}{c^2} = \frac{4.19 \times 10^{-14}\ \cancel{\text{J}} / \text{m}^3}{9.0 \times 10^{16}\ \cancel{\text{J}} / \text{kg}} = \frac{4.19}{9.0} \times 10^{-30}\ \text{kg} / \text{m}^3$$

$$\boxed{\rho_m = 4.66 \times 10^{-31}\ \text{kg} / \text{m}^3}$$

Lastly, we determine what fraction of the critical density (ρ_{crit}) this value represents. Recall from the question that $\rho_{crit} = 10^{-26}$ kg/m^3. Using this value, find:

$$\frac{\rho_m}{\rho_{crit}} = \frac{4.66 \times 10^{-31}\ \cancel{\text{kg} / \text{m}^3}}{10^{-26}\ \cancel{\text{kg} / \text{m}^3}} \quad \rightarrow \quad \boxed{\frac{\rho_m}{\rho_{crit}} = 4.66 \times 10^{-5}}$$

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4) The star Phi Orionis, like the Sun, is powered by the fusion of Hydrogen to Helium. The mass of Phi Orionis is $M_{phi} = 18\, M_{sun}$. The luminosity of Phi Orionis is $L_{phi} = 20,000\, L_{sun}$. Discuss the likelihood of intelligent life existing on a planet orbiting the star Phi Orionis. [Questions you might want to consider: What is the "life span" of Phi Orionis? How long did it take intelligent life to develop on Earth? How far would you have to be from Phi Orionis to receive the same flux of light that we receive here on Earth from the Sun?]

For this question, it helps to calculate a few quantities in advance to motivate possible responses. In the spirit of the preceding questions, start with the "life span" of Phi Orionis. Following the example of question 2, find:

$$(\, t_{phi} \,/\, t_{sun} \,) = \frac{M_{phi} \,/\, M_{sun}}{L_{phi} \,/\, L_{sun}} = \frac{18}{20,000} = 9 \times 10^{-4} \quad \rightarrow \quad \boxed{t_{phi} = 9 \times 10^{-4} \cdot t_{sun} = 9 \text{ million years}}$$

Next, given luminosity, determine the orbital distance where flux would be similar to what Earth receives from the Sun. Recall that flux is expressed as $f = L/(4\pi r^2)$. We can thus write:

$$f_{phi} = L_{phi}/(4\pi r_{phi}^2) \quad \text{and} \quad f_{sun} = L_{sun}/(4\pi r_{sun}^2)$$

Setting the two fluxes equal, find:

$$L_{phi}/(4\pi r_{phi}^2) = L_{sun}/(4\pi r_{sun}^2) \quad \rightarrow \quad \frac{L_{phi}}{L_{sun}} = \frac{\cancel{4\pi}\, r_{phi}^2}{\cancel{4\pi}\, r_{sun}^2} \quad \rightarrow \quad r_{phi} = r_{sun} \cdot \sqrt{\frac{L_{phi}}{L_{sun}}}$$

Plugging in $L_{phi} \,/\, L_{sun} = 20,000$ and $r_{sun} = 1\text{AU}$, find:

$$r_{phi} = 1 \text{ AU} \cdot \sqrt{20,000} \approx \boxed{141 \text{ AU} = r_{phi}}$$

The likelihood of finding life around a star such as Phi Orionis is pretty slim. Although there is no exact answer, we can make several observations based on our experience here on Earth. I'll first address the suggested questions and how those relate to the development of intelligent life. I'll then finish with some other points you might have mentioned that are also relevant. Read on to see some of the the major points and their associated explanations.

Intelligent life requires time to develop. Above, we found a 9-million-year lifespan for Phi Orionis. This would pose significant challenges to the development of intelligent life.

The solar system (Sun and planets) is roughly 4.5 billion years old. During its infancy, the solar system (including Earth) was a fairly inhospitable place. Frequent collisions with asteroids, comets, and the such continuously resurfaced the terrestrial planets. The fossil record seems to indicate that primitive live began more or less right away (millions or tens of millions of years) after this constant bombardment ceased, roughly 3.5 billion years ago. This suggests that such primitive organisms are fairly easy to form (at least in our case). At the same time, complex multicellular life ("intelligent") did not appear until far more recently (within the past billion years). This suggests that intelligent life requires substantially more time to emerge.

The computed lifespan of 9 million years for Phi Orionis is not compatible with this life emergence timeline. In reality, it probably took of order 10 million years to actually *form* the Earth in the first place. Assuming Phi Orionis has not exhausted its fuel before the planet fully forms, it will almost certainly have done so well before debris bombardment ceases. That a long time baseline of a few billion years might be required to evolve intelligent life makes its existence even less plausible in such a solar system.

Reasonable conditions are required for life. As stated in the problem, Phi Orionis is an extremely luminous star which poses potential problems for the development of life as it occurred on Earth.

Life as we know it has requirements. Chief among these is the need for liquid water, which only exists within a range of temperatures. The temperature of a planet is primarily determined by the energy flux from its host star. In order to ensure that water can exist in liquid phase, the stellar flux on the planet should be comparable to what we receive on Earth (so that the temperatures are roughly the same). The range of orbital distances that permit liquid water is typically known as the "habitable zone."

Above we computed that a planet orbiting Phi Orionis at a distance of roughly 141 AU would receive the same flux as Earth does from the Sun. This would ideally result in an equilibrium temperature where water could persist in liquid form. It is important to note that 141 AU is quite far away from the central star. Although this isn't certain (still an area of active research), it is probably very difficult to form a usefully-sized planet that far from the central star. For instance, in our solar system, the rocky planets (those that were, are, or could be habitable) all reside in the inner solar system. The outer planets are all gas or ice giants which would be far less conducive to life.

Hotter stars → sunburn. As previously stated, Phi Orionis is a very luminous star. Among normal (not giant) stars, more luminous stars are also hotter. Recall from earlier in the course that the temperature dictates what wavelengths of light are emitted (blackbodies). Hotter stars emit light at shorter wavelengths (bluer light). Phi Orionis has a temperature close to 30,000 K, far hotter than our Sun (5800 K). This far higher surface temperature means that Phi Orionis produces a great deal more high-energy UV light than our sun does. This UV radiation can easily damage organic molecules (e.g., sunburn) and could easily suppress surface life on any planets it hosts.