# 1 Monday, October 3: Stellar Atmospheres

There exist entire books written about stellar atmospheres; I will only give a brief sketch of the simplest approximations used in studying stellar atmospheres. In particular, I want to discuss that most useful quantity, the *Rosseland mean opacity*.[1]

A photon of frequency $\nu$ travels an infinitesimal distance $ds$ inside a star. In that distance, it can be scattered or absorbed. For simplicity, I will assume that the scattering is coherent, and that the energy of an absorbed photon re-emerges as thermal radiation. The radiative transfer equation for scattering is

$$\frac{dI_\nu}{ds}(\text{scattering}) = -\sigma_\nu I_\nu + \sigma_\nu J_\nu \ , \tag{1}$$

where $\sigma_\nu$ is the scattering coefficient and $J_\nu$ is the angle-averaged specific intensity.[2] The radiative transfer equation for absorption is

$$\frac{dI_\nu}{ds}(\text{absorption}) = -\alpha_\nu I_\nu + \alpha_\nu S_\nu \ , \tag{2}$$

where $\alpha_\nu$ is the absorption coefficient, and the source function $S_\nu$ is equal to the Planck function $B_\nu(T)$ if, as we've assumed, the energy of the absorbed photons is converted to thermal radiation.

Combining the effects of absorption and scattering, we find

$$\frac{dI_\nu}{ds} = -(\alpha_\nu + \sigma_\nu)I_\nu + (\alpha_\nu B_\nu + \sigma_\nu J_\nu) \ . \tag{3}$$

This equation can be converted to a simpler form

$$\frac{dI_\nu}{ds} = -(\alpha_\nu + \sigma_\nu)(I_\nu - S_\nu) \tag{4}$$

if we define a source function

$$S_\nu \equiv \frac{\alpha_\nu B_\nu + \sigma_\nu J_\nu}{\alpha_\nu + \sigma_\nu} \ , \tag{5}$$

[1] A search in astro-ph, for instance, reveals an ongoing conflict between the Rosseland mean opacity computed by the Opacity Project and that computed by the OPAL group. Why do these people care so intensely about the Rosseland mean opacity? I hope to explain in this lecture.

[2] Note that if the specific intensity $I_\nu$ is isotropic to begin with, coherent scattering doesn't affect it.

which is just the mean of the Planck function and the angle-averaged specific intensity, weighted by the absorption coefficient and the scattering coefficient, respectively.

In general, solving the radiative transfer equation is a tedious chore. However, in studying stellar atmospheres, some simplifying assumptions can be made. The outer layers of a star are relatively low in density, so radiation is the only significant method of transporting energy.[3] Moreover, gradients in temperature and specific intensity are much greater in the radial direction than in the transverse directions. Thus, a stellar atmosphere can usually be well approximated as a *plane parallel* system, in which properties of the atmosphere, such as $T$, $\sigma_\nu$, and $\alpha_\nu$ depend only on the vertical coordinate $z$.[4]

In the plane parallel approximation, the specific intensity $I_\nu$ depends only on $z$ and on the angle $\theta$ between the direction of the light ray and the $z$ axis: when the light travels straight upward, in the same direction that $z$ increases, $\theta = 0$; when the light travels straight down, $\theta = \pi$. In traversing a thin layer of the atmosphere, of vertical thickness $dz$, the distance traveled by light is

$$ds = \frac{dz}{\cos\theta} \ , \tag{6}$$

implying $(ds)^2 \geq (dz)^2$. The radiative transfer equation in a plane parallel atmosphere thus takes the form

$$\cos\theta \frac{\partial I_\nu(z,\theta)}{\partial z} = -[\alpha_\nu(z) + \sigma_\nu(z)][I_\nu(z,\theta) - S_\nu(z)] \ . \tag{7}$$

I have assumed that the emission is thermal, and that the scattering is coherent. The above equation can be rewritten as

$$I_\nu = S_\nu - \frac{\cos\theta}{\alpha_\nu + \sigma_\nu} \frac{\partial I_\nu}{\partial z} \ . \tag{8}$$

Note that when $\cos\theta = 0$, indicating that a light ray is running parallel to the star's surface, the specific intensity $I_\nu$ is equal to the source function $S_\nu$.

[3]In a star's atmosphere, you don't have to worry about the messy details of convective heat transport. Hurray!

[4]When the Earth's atmosphere is treated with a plane parallel approximation, it is sometimes called a "flat Earth" model. Like a spherical cow, a flat planet or star is sometimes a useful approximation. (And sometimes not.)

When $\cos\theta \neq 0$, there is a correction term proportional to the gradient of $I_\nu$ in the $z$ direction.

How can we solve equation (8) for light traveling at an arbitrary angle $\theta$ in a real star's atmosphere? To begin with, let's make the assumption that the atmosphere is homogeneous and isotropic; this will be our zeroth order approximation. In this approximation, $I_\nu^{(0)} = J_\nu^{(0)}$ and, from equation (8),

$$I_\nu^{(0)} = S_\nu^{(0)} \ . \tag{9}$$

However, the definition of the source function $S_\nu$ (equation 5) tells us that

$$(\alpha_\nu + \sigma_\nu) S_\nu = \alpha_\nu B_\nu + \sigma_\nu J_\nu \ , \tag{10}$$

so

$$I_\nu^{(0)} = B_\nu \ . \tag{11}$$

Thus, a homogeneous isotropic atmosphere in which the emission is thermal produces a blackbody spectrum. Now let's take into account the fact that the specific intensity isn't perfectly homogeneous, but has a gradient in the $z$ direction. By plugging our zeroth order approximation, $I_\nu^{(0)} = B_\nu$, back into equation (8), we find that

$$I_\nu^{(1)} = B_\nu - \frac{\cos\theta}{\alpha_\nu + \sigma_\nu} \frac{\partial B_\nu}{\partial z} \ . \tag{12}$$

Note that if the temperature $T$ decreases as you go upward in the atmosphere, then the upward specific intensity ($\cos\theta = 1$) is slightly greater than it would be in a homogeneous atmosphere, and the downward specific intensity ($\cos\theta = -1$) is slightly less. This means that there is a net *upward* flux of light energy in this case.[5]

The net flux $F_\nu$ through a thin layer of the atmosphere can be found by doing the usual integration over angle (see the notes for Friday, September 23):

$$F_\nu(z) = \int I_\nu^{(1)}(z, \theta) \cos\theta d\Omega = 2\pi \int_{-1}^{1} I_\nu^{(1)}(z, \theta) \cos\theta d(\cos\theta) \ . \tag{13}$$

The homogeneous component of the specific intensity contributes nothing to the net flux; the upward flux exactly balances the downward flux. Only the

[5] In other words, energy flows from regions of higher temperature to regions of lower temperature. This is reassuringly consistent with the second law of thermodynamics.

component proportional to the gradient of $B_\nu$ makes a contribution:

$$F_\nu(z) = -\frac{2\pi}{\alpha_\nu + \sigma_\nu}\frac{\partial B_\nu}{\partial z}\int_{-1}^{1}\cos^2\theta d(\cos\theta) \ . \tag{14}$$

Performing the integral (which has a value of 2/3), we find

$$F_\nu(z) = -\frac{4\pi}{3(\alpha_\nu + \sigma_\nu)}\frac{\partial B_\nu}{\partial T}\frac{\partial T}{\partial z} \ . \tag{15}$$

(This first order approximation for the flux of light energy within a star is known as the *diffusion approximation*, since it has the same functional form as the equation that describes the diffusion of molecules in a gas.)

Since we know the functional form for the Planck function $B_\nu(T)$, the flux $F_\nu$ can be computed for each value of $z$ if we know $T$, $\alpha_\nu$, and $\beta_\nu$ as functions of $z$. For instance, if you're dealing with low-frequency radiation, $h\nu \ll kT$, you can use the Rayleigh-Jeans approximation: $B_\nu \approx (2\nu^2/c^2)kT$, which yields

$$F_\nu(z) \approx -\frac{8\pi\nu^2}{3c^2(\alpha_\nu + \sigma_\nu)}k\frac{\partial T}{\partial z} \ . \tag{16}$$

In real stars, it must be admitted, the effective absorption coefficient, $\alpha_\nu + \sigma_\nu$ can be a complicated function of frequency, showing the effects of various atomic absorption lines. If you just want to know the total flux integrated over all frequencies, you may write

$$F(z) = \int_0^\infty F_\nu(z)d\nu = -\frac{4\pi}{3}\frac{\partial T}{\partial z}\int_0^\infty \frac{1}{\alpha_\nu + \sigma_\nu}\frac{\partial B_\nu}{\partial T}d\nu \ . \tag{17}$$

When Svein Rosseland, back in the 1920s, looked at an equation of this sort, he realized that it could be rewritten in the form

$$F(z) = -\frac{4\pi}{3}\frac{\partial T}{\partial z}\frac{1}{\alpha_R}\int_0^\infty \frac{\partial B_\nu}{\partial T}d\nu \ , \tag{18}$$

where

$$\frac{1}{\alpha_R} \equiv \frac{\int_0^\infty (\alpha_\nu + \sigma_\nu)^{-1}(\partial B_\nu/\partial T)d\nu}{\int_0^\infty (\partial B_\nu/\partial T)d\nu} \ . \tag{19}$$

The quantity $\alpha_R$ defined in this manner is called the *Rosseland mean absorption coefficient*, in honor of Professor Rosseland. Why on earth would you bother to define an effective absorption coefficient in this way, by weighting the inverse of $\alpha_\nu + \sigma_\nu$ with a weighting function $\partial B_\nu / \partial T$ and then integrating? Well, note that

$$\int_0^\infty \frac{\partial B_\nu}{\partial T} d\nu = \frac{\partial}{\partial T} \partial_0^\infty B_\nu d\nu = \frac{\partial}{\partial T} \frac{1}{\pi \sigma T^4} = \frac{4\sigma T^3}{\pi} \ . \tag{20}$$

Thus, equation (18) can be written in the (relatively) simple form

$$F(z) = -\frac{16\sigma T^3}{3\alpha_R} \frac{\partial T}{\partial z} \ . \tag{21}$$

In practice, if you want to build a model stellar atmosphere, you look up the appropriate Rosseland mean opacity for the appropriate temperature, density, and chemical composition. The Rosseland mean opacity, $\kappa_R$, is equal to

$$\kappa_R = \alpha_R / \rho \ , \tag{22}$$

where $\rho$ is the mass density. Thus, the opacity $\kappa_R$ has units of $\mathrm{cm}^2\,\mathrm{g}^{-1}$. Figure 1 shows the Rosseland mean opacity as a function of temperature and

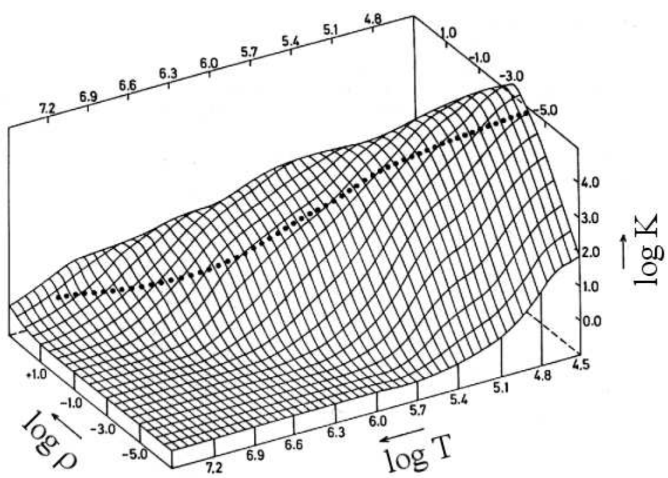


Figure 1: Rosseland mean opacity as a function of $T$ and $\rho$.

density for gas of solar metallicity. The dotted line represents the Rosseland mean opacity in the Sun, from the hot, high density interior to the cooler, lower density photosphere.

## 2 Wednesday, October 5: Plane Waves

Light can be regarded as waves that propagate in electric and magnetic fields. The ancient Greeks were vaguely aware of electricity; the story goes that the philosopher Thales rubbed a piece of amber with fur, and was able to pick up bits of straw and dead leaves.[6] The ancient Greeks were also vaguely aware of magnetism; the story goes that a shepherd in Magnesia (a region in Thessaly) became aware that lumps of ore (of the type now known as magnetite) attracted the iron nails in his boot soles.[7] For centuries, electricity and magnetism were regarded as mysterious properties of a few selected substances like amber and magnetite; they were also thought to be unrelated to each other.

In 1820, Oersted demonstrated that an electric current was capable of deflecting a magnetized compass needle; shortly afterward, Faraday showed that a moving magnet produced an electric field. Half a century later, James Clerk Maxwell put forward a mathematically based electromagnetic theory. Despite the intervening upheavals of relativity and quantum mechanics, Maxwell's equations are still useful today. At every point $\vec{r}$, at every time $t$, according to Maxwell, there exists a *electric field strength* $\vec{E}(\vec{r}, t)$ and a *magnetic flux density* $\vec{B}(\vec{r}, t)$. If a small particle (a bit of amber, for instance) has an electric charge $q$ and is moving with a velocity $\vec{v}$, it experiences a force

$$\vec{F} = q\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right) . \tag{23}$$

(Here, I have assumed $v \ll c$.) Equation (23) states that the force exerted by the magnetic field is perpendicular to the velocity vector; the rate at which work is done on the charged particle is thus

$$\vec{F} \cdot \vec{v} = q\vec{E} \cdot \vec{v} . \tag{24}$$

We'll follow Rybicki and Lightman in using *Gaussian units* for electric charge. In Gaussian units, the electric field of a point particle with charge $q$ is

$$\vec{E} = \frac{q}{r^2}\frac{\vec{r}}{r} . \tag{25}$$

[6]The word "electricity" comes from the ancient Greek word for amber.

[7]The word "magnetism" comes from the geographical region of Magnesia.

Thus, identical particles will repel each other with a force

$$F = \frac{q^2}{r^2} \ . \tag{26}$$

In the Gaussian cgs system, the basic unit of electric charge is called the "electrostatic unit", or *esu*, for short. It is the amount of electric charge such that two particles, each with a charge of 1 esu, repel each other with a force of 1 dyne when placed 1 cm apart.[8] Thus, equation (26) really should be more accurately written as

$$F = \left[ 1 \frac{\mathrm{g\,cm^3\,s^{-2}}}{\mathrm{esu^2}} \right] \frac{q^2}{r^2} \ . \tag{27}$$

In practice, it is easier to leave out the term in square brackets by stating

$$1\,\mathrm{esu} = 1\,\mathrm{g^{1/2}\,cm^{3/2}\,s^{-1}} \ . \tag{28}$$

It may feel a bit strange to measure electric charge in these units, but it makes relations such as equations (23) and (26) look a lot simpler! By defining the esu in this way, the units in which $E$ and $B$ are measured become $\mathrm{g^{1/2}\,cm^{-1/2}\,s^{-1}}$.[9]

The electric charge is quantized, but any macroscopic bit of material will contain a very, very large number of electrons and protons. Thus, at any point $\vec{r}$, you can estimate a charge density $\rho_q$ and an electric current $\vec{j}$ by constructing a box centered on the point $\vec{r}$ large enough to contain many charged particles, but still small compared to the total system at which you are looking. If the volume of the box is $V$, and it contains $N \gg 1$ charged particles, the charge density is

$$\rho_q = \frac{1}{V} \sum_{i=1}^{N} q_i \tag{29}$$

and the electric current is

$$\vec{j} = \frac{1}{V} \sum_{i=1}^{N} q_i \vec{v}_i \ . \tag{30}$$

[8]The charge of an electron is $q_e = -4.80 \times 10^{-10}$ esu; the choice of sign is due to Benjamin Franklin.

[9]For magnetic fields, the basic cgs unit of magnetic flux density is called the "gauss"; the basic unit of electric field strength doesn't have a special name – it's just "dyne/esu".

The complete set of Maxwell's equations is

$$\begin{aligned} \vec{\nabla} \cdot \vec{D} &= 4\pi\rho_q && (31) \\ \vec{\nabla} \cdot \vec{B} &= 0 && (32) \\ \vec{\nabla} \times \vec{E} &= -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} && (33) \\ \vec{\nabla} \times \vec{H} &= \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{D}}{\partial t} \ . && (34) \end{aligned}$$

Here the quantity $\vec{D}$ is the *electric displacement*, related to the electric field strength by the relation $\vec{D} = \epsilon\vec{E}$, where $\epsilon$ is the dielectric constant. The quantity $\vec{H}$ is the *magnetic field strength*, related to the magnetic flux density by the relation $\vec{B} = \mu\vec{H}$, where $\mu$ is the magnetic permeability. In general, the dielectric constant and magnetic permeability of a substance is not equal to one; however, in a vacuum, $\epsilon = \mu = 1$, and you don't have to worry about the distinction between $\vec{D}$ and $\vec{E}$, or between $\vec{B}$ and $\vec{H}$.

In a vacuum, Maxwell's equations take the simpler form

$$\begin{aligned} \vec{\nabla} \cdot \vec{E} &= 0 && (35) \\ \vec{\nabla} \cdot \vec{B} &= 0 && (36) \\ \vec{\nabla} \times \vec{E} &= -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} && (37) \\ \vec{\nabla} \times \vec{B} &= \frac{1}{c}\frac{\partial \vec{E}}{\partial t} \ . && (38) \end{aligned}$$

The interesting thing about these equations is that they permit a solution that consists of waves moving through space. If we take the curl of equation (38), we have

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \frac{1}{c}\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{E}) \ . \quad (39)$$

Substituting for the curl of $\vec{E}$ from equation (37), we find

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = -\frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2} \ . \quad (40)$$

However, using the vector identity $\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{B}) - \vec{\nabla}^2\vec{B}$, and taking advantage of the fact that $\vec{\nabla} \cdot \vec{B} = 0$, we find the result is a wave equation

$$\vec{\nabla}^2\vec{B} = \frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2} \ , \quad (41)$$

Figure 2: Of course, Maxwell's equations can be written in different forms.

representing wave propagation at a speed $c$. An identical equation is found for the electric field strength $\vec{E}$.

For illustrative purposes, let's examine the case of sinusoidal plane waves propagating along the $x$ axis:

$$\vec{B} = \hat{e}_B B_0 e^{i(kx-\omega t)} \tag{42}$$

$$\vec{E} = \hat{e}_E E_0 e^{i(kx-\omega t)} \ , \tag{43}$$

where $B_0$ and $E_0$ are real numbers. Substitution back into the wave equation confirms that the wavenumber $k = 2\pi/\lambda$ and the angular frequency $\omega = 2\pi\nu$ are linked together by the relation $ck = \omega$, or $c/\lambda = \nu$. Maxwell's equations for these electric and magnetic waves are

$$ik(\hat{e}_x \cdot \hat{e}_E) E_0 e^{i(kx-\omega t)} = 0 \tag{44}$$

$$ik(\hat{e}_x \cdot \hat{e}_B) B_0 e^{i(kx-\omega t)} = 0 \tag{45}$$

$$ik(\hat{e}_x \times \hat{e}_E) E_0 e^{i(kx-\omega t)} = i\frac{\omega}{c} \hat{e}_B B_0 e^{i(kx-\omega t)} \tag{46}$$

$$ik(\hat{e}_x \times \hat{e}_B) B_0 e^{i(kx-\omega t)} = i\frac{\omega}{c} \hat{e}_E E_0 e^{i(kx-\omega t)} \ . \tag{47}$$

Equations (44) and (45) show us that the vectors $\hat{e}_B$ and $\hat{e}_E$ are both perpendicular to the $x$ axis; that is, the axis along which the wave is propagating. Equation (46) shows us that the vector $\hat{e}_B$ is perpendicular to the plane defined by the vectors $\hat{e}_E$ and $\hat{e}_x$. That is, the vectors $\hat{e}_B$, $\hat{e}_E$, and $\hat{e}_x$ form a set of mutually perpendicular cartesian axes. Equations (46) and (47) tell us that $E_0 = B_0$, and also confirm that the frequencies and phases of the $\vec{E}$ wave and the $\vec{B}$ wave *must* be identical (see Figure 3).

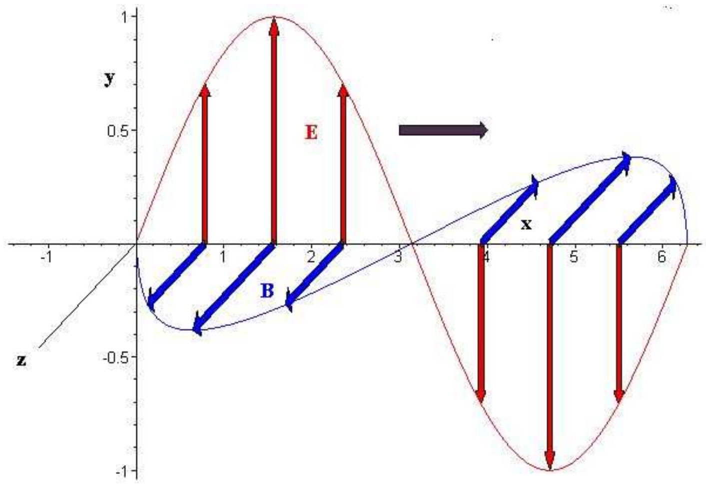


Figure 3: Electromagnetic wave moving along the $x$ axis ($E_0 = B_0 = 1$).

# 3 Friday, October 7: Polarization

In the previous lecture, I discussed a simple monochromatic plane wave of light. For such a wave, I can choose a coordinate system such that

$$\vec{E} = \hat{e}_y E_0 \cos(kx - \omega t) \tag{48}$$

and

$$\vec{B} = \hat{e}_z E_0 \cos(kx - \omega t) \ . \tag{49}$$

Such a wave is *linearly polarized.* A wave is linearly polarized if its electric field strength $\vec{E}$ always remains in the same plane. Suppose we wanted to observe the electric field strength in a plane perpendicular to the direction of motion of the wave; let's choose the $x = 0$ plane for mathematical simplicity. For the wave described in equation (48), the time variation of $\vec{E}$ in the $x = 0$ plane is

$$\vec{E} = \hat{e}_y E_0 \cos(-\omega t) = \hat{e}_y E_0 \cos(\omega t) \ . \tag{50}$$

That is, $\vec{E}$ just oscillates up and down along the $y$ axis; not very thrilling.

However, a monochromatic wave doesn't have to display linear polarization. Consider, for instance, the superposition of two monochromatic plane waves. One has the electric field strength

$$\vec{E}_1 = \hat{e}_y E_0 \cos(kx - \omega t) \ , \tag{51}$$

and the other has electric field strength

$$\vec{E}_2 = \hat{e}_z E_0 \cos(kx - \omega t + \phi) \ . \tag{52}$$

(We don't have to worry our heads about the corresponding magnetic flux densities; Maxwell's equations instantly dictate what they must be.) The two waves have the same wavenumber $k$ and the same amplitude $E_0$, but have a phase shift of $\phi$ relative to each other. In the $x = 0$ plane, $\vec{E}_1$ varies along the $y$ axis, with time dependence

$$E_y = E_0 \cos(-\omega t) = E_0 \cos \omega t \ . \tag{53}$$

In the $x = 0$ plane, $\vec{E}_2$ varies along the $z$ axis, with time dependence

$$E_z = E_0 \cos(-\omega t + \phi) = E_0 [\cos\phi \cos\omega t + \sin\phi \sin\omega t] \ . \tag{54}$$

How does the vector $\vec{E} = E_y \hat{e}_y + E_z \hat{e}_z$ vary with time in the $x = 0$ plane? Well, the angle $\theta$ between $\vec{E}$ and the $y$ axis is given by the relation

$$\tan\theta = \frac{E_z}{E_y} = \frac{\cos\phi\cos\omega t + \sin\phi\sin\omega t}{\cos\omega t} = \cos\phi + \sin\phi\tan\omega t \ . \tag{55}$$

Thus, if $\sin\phi \neq 0$, the electric field strength vector $\vec{E}$ will rotate in the $x = 0$ plane. The length of $\vec{E}$ is given by the relation

$$\begin{aligned} &|\vec{E}|^2 = E_y^2 + E_z^2 = \\ &E_0^2 [(1 + \cos^2\phi)\cos^2\omega t + 2\cos\phi\sin\phi\cos\omega t\sin\omega t + \sin^2\phi\sin^2\omega t] \ . \end{aligned} \tag{56}$$

Let's look at two special cases.

Suppose that the two plane waves are in phase: $\phi = 0$. This implies, from equation (55),

$$\tan\theta = 1 \ , \tag{57}$$

or $\theta = \pi/4$. That is, the $\vec{E}$ vector points at a 45 degree angle, tilted midway between the $y$ and $z$ axis. The length of the $\vec{E}$ vector, from equation (57), is given by the relation

$$|\vec{E}|^2 = E_0^2 [2\cos^2\omega t] \ . \tag{58}$$

Thus, if the two plane waves are in phase with each other, they add together to create a linearly polarized wave, of amplitude $\sqrt{2}E_0$, tilted midway between the two component plane waves.

Now suppose that the two plane waves are out of phase by $\phi = \pi/2$. In this case, the angle of the $\vec{E}$ vector in the $x = 0$ plane is given by the relation (equation 55)

$$\tan\theta = \tan\omega t \ . \tag{59}$$

That is, $\theta = \omega t$, and the $\vec{E}$ vector in the $x = 0$ plane rotates with a constant angular frequency $\omega$. The length of the $\vec{E}$ vector is given by the relation (equation 57)

$$|\vec{E}|^2 = E_0^2[\cos^2\omega t + \sin^2\omega t] = E_0^2 \ . \tag{60}$$

That is, the electric field strength vector in the $x = 0$ plane (or in any plane perpendicular to the wave's motion) simply goes around and around and around in a circle at constant angular frequency. A wave that shows this type of behavior is thus referred to as having *circular polarization.*[10] At a given time $t$, the curve traced out by the vector $\vec{E}$ of a circularly polarized wave is a helix (Figure 4).

If the phase $\phi$ has an arbitrary value, it can be shown that $\vec{E}$ traces out an ellipse in the $x = 0$ plane, with circular polarization and linear polarization being the special limiting cases of *elliptical polarization*. Playing with polarized light in the laboratory is lots of fun, but producing polarized light seems to require birefringent substances (like calcite crystals), dichroic substances (like Polaroid filters), or reflection from dielectric substances at the Brewster angle.[11] Under what circumstances is polarization important in the universe at large?

To begin with, thermal radiation is not polarized. In the random motions that characterize a hot substance, there are no preferred orientations. A blackbody, then, emits light of all wavelengths, all directions of motion, and all phases. As a result, light from a blackbody is unpolarized. However,

[10] Parenthetical comment: if $\phi = \pi/2$, then $\theta = \omega t$, and $\vec{E}$ rotates counterclockwise as seen from the positive $x$ axis; this is conventionally called "right-handed" circular polarization. If $\phi = -\pi/2$, then $\theta = -\omega t$, and $\vec{E}$ rotates clockwise; this is called "right-handed" circular polarization.

[11] I am just throwing in terms like "birefringent", "dichroic", and "Brewster angle" to add artistic verisimilitude; discussing them in detail would take us far astray from the subject of the course.

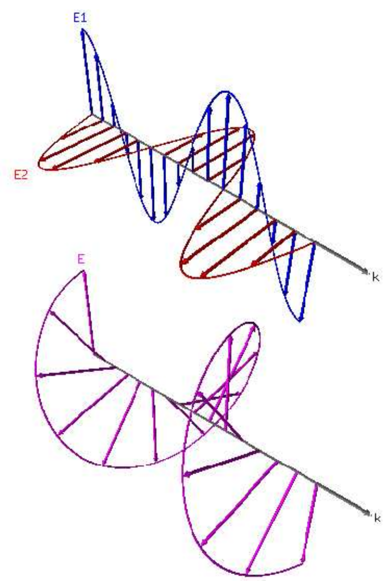


Figure 4: Circular polarization as the result of superimposed linearly polarized waves.

when electrons are accelerated by a magnetic field, there is a preferred direction, dictated by the magnetic flux density $\vec{B}$. Cyclotron and synchrotron radiation, therefore, can be polarized under some conditions. Later in the course, when we discuss synchrotron emission in some detail, we'll examine the possible polarization of synchrotron radiation.

In view of the fact that thermal emission is unpolarized, it is remarkable to note that the light from stars near the plane of our galaxy is observed to be polarized (Figure 5) at a level of a few percent. So what's happening here? Even if light is unpolarized when it is created in a star's photosphere, it can achieve polarization by its scattering interactions. It is found that the most highly polarized starlight is also the most reddened starlight; this indicates that the polarization is due to interactions with interstellar dust grains, which also redden the starlight.

Dust grains, in general, are not spherical (Figure 6). Consider a single dust grain, which we may approximate as a triaxial ellipsoid with semimajor axes of length $a \geq b \geq c$. The scattering of a linearly polarized light wave by the grain is most efficient when $\vec{E}$ lies along the longest axis of the grain. It is least efficient when $\vec{E}$ is aligned with the shortest axis. Thus, if the long axes of the interstellar dust grains are preferentially perpendicular to the plane of

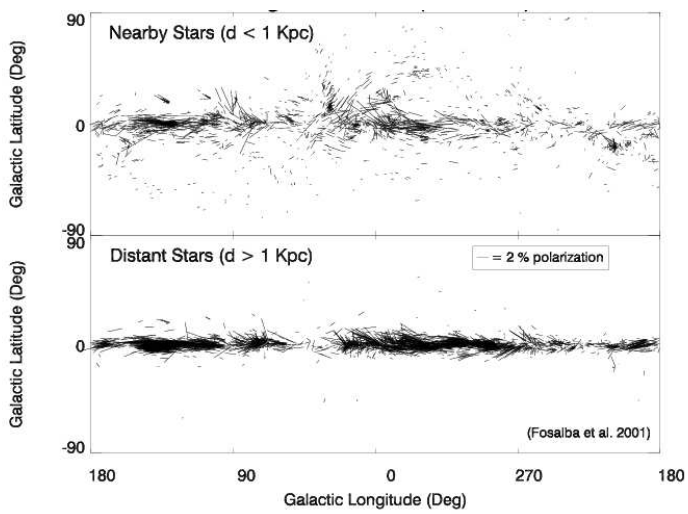


Figure 5: Polarization of starlight: sample of 5500 stars in our galaxy. Each line indicates the direction of $\vec{E}$; its length is proportional to the percentage polarization.

the Milky Way, waves with $\vec{E}$ running north-south will be suppressed relative to waves with $\vec{E}$ running east-west, along the Milky Way. This would produce the preferential polarization shown in Figure 5.

However, what causes dust particles to be preferentially oriented with their long axes perpendicular to the plane of our galaxy? After all, they are constantly being bombarded by gas molecules and photons. Each collision imparts a torque to the grain. The estimated spin angular frequency for the tiny grains in the interstellar medium is typically $\omega_{\rm grain} \sim 10^5\,{\rm s}^{-1}$, or even greater. Various means for orienting these rapidly spinning dust grains have been proposed. They all make use of the interstellar magnetic field $\vec{B}$, which is thought to have a typical value, in cgs units, of $B \sim 10^{-6}\,$gauss. Dust grains are made largely of *paramagnetic* materials; that means if you place them in an external magnetic field $\vec{B}$, they develop an internal magnetic field $\vec{B}_{\rm int}$ that is parallel to $\vec{B}$. However, the alignment of the internal field with the external field is not instantaneous; thus, for a grain rotating about an axis that is not parallel to the external field $\vec{B}$, $\vec{B}_{\rm int}$ will be slightly misaligned with $\vec{B}$. The interaction of the internal and external field causes a torque which acts to slow the rotation of the grain. The end state of the grain is rotation about its short axis, with this axis pointing in the direction of $\vec{B}$; in

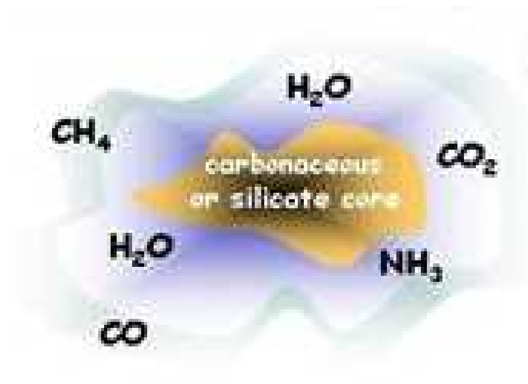


Figure 6: Typical elongated interstellar dust grain.

this state, $\vec{B}_{\rm int}$ remains aligned with $\vec{B}$. The long axis, as the grain continues to spin, stays in the plane perpendicular to $\vec{B}$.

Our picture, therefore, is of a galactic magnetic field $\vec{B}$ that lies in the plane of the Milky Way; dust grains spin around with their long axes perpendicular to the magnetic field $\vec{B}$, and the linear polarization of light scattering from the dust grains tends to lies parallel to $\vec{B}$.