10. Supermassive Black Holes

The Innermost Stable Circular Orbit

Consider an object (planet, rock, atom) in a circular orbit around an object of mass M.

In Newtonian gravity, nudging this object slightly will send it into a slightly elliptical orbit.

At distances $r \gg R_{\rm Sch} = 2GM/c^2$, GR is similar to Newtonian gravity, so the same is true.

But close to $R_{\rm Sch}$, the inward pull of gravity in GR is stronger than it would be in Newtonian gravity.

Describing orbits in this regime properly requires the GR description of spacetime curvature, but it's a surprisingly good approximation to use Newtonian gravity but change the force law from $F = GMm/r^2$ to

$$F_{\rm Pseudo-Newtonian} = \frac{GMm}{(r - R_{\rm Sch})^2} \ .$$

A key difference between GR and Newtonian gravity: for $r < 3R_{\rm Sch}$, a slight nudge will send an object spiraling into the black hole instead of going into an elliptical orbit.

We refer to $3R_{\rm Sch}$ as the radius of the Innermost Stable Circular Orbit (ISCO).

Accretion disks have a "hole" inside the ISCO, since gas that enters this zone quickly spirals into the black hole's event horizon.

Gas at the ISCO is orbiting at roughly half the speed of light, so EM radiation emitted from the inner zone of the accretion disk (typically X-rays or ultraviolet light) can be strongly redshifted (gas moving away from us) and blueshifted (gas coming toward us).

In addition, this EM radiation experiences a strong gravitational redshift as it is coming from only a few $R_{\rm Sch}$.

For a spinning black hole, the swirl of spacetime helps support objects against the pull of gravity, so the radius of the ISCO is smaller (between 1 and 3 $R_{\rm Sch}$).

The Doppler shifts and gravitational redshift can be even more extreme in this case.

Luminosity of an Accreting Black Hole

The luminosity of an accreting black hole is proportional to the rate at which it is gaining mass.

A non-spinning black hole accreting gas at a rate $\dot{M}$ through a thin disk has a luminosity

$$L \approx \frac{1}{12}\dot{M}c^2.$$

Note that $\dot{M}$ is a rate of change of mass, with units of, for example, $M_\odot {\rm yr}^{-1}$ (or ${\rm kg\,s}^{-1}$). Mc^2 is energy, so $\dot{M}c^2$ is energy/time, the units of luminosity (or power).

The factor 1/12 comes from calculating the amount of gravitational potential energy gained by the gas as it moves from a large distance to the ISCO $3R_{\rm Sch}$, after which it spirals in without radiating more energy.

For a spinning black hole, the gas can get closer before it spirals in, so it can gain more energy and the factor in this equation would be bigger than 1/12.

However, if there is too little "friction" in the flow of accreting gas, then it will just heat up instead of radiating the energy that it gains, and it may eventually carry that heat energy with it into the black hole.

In this case we would get a "puffy" disk with lower luminosity.

We can plug in numbers to scale this equation to a form that is useful for either stellar mass black holes or supermassive black holes.

Use the fact that $1M_\odot \mathrm{yr}^{-1} = 2 \times 10^{30}\,\mathrm{kg}/3.16 \times 10^7\mathrm{s} = 6.33 \times 10^{22}\,\mathrm{kg/s}$ and $1L_\odot = 4 \times 10^{26}\,\mathrm{joule/s}$.

With these numbers we can write

$$L = 1.2 \times 10^5 L_\odot \times \left(\frac{\dot{M}}{10^{-7} M_\odot \mathrm{yr}^{-1}} \right) ,$$

or

$$L = 1.2 \times 10^{12} L_\odot \times \left(\frac{\dot{M}}{1 M_\odot \mathrm{yr}^{-1}} \right) .$$

A non-spinning stellar mass black hole that accretes $1M_\odot$ over 10 million years from a companion has a luminosity of $1.2 \times 10^5 L_\odot$.

A non-spinning supermassive black hole that is accreting $1M_\odot$ per year at the center of a galaxy has a luminosity of 1.2 trillion solar luminosities, which can easily exceed the luminosity of all the galaxy's stars put together.

The Eddington Luminosity Limit

The mass of the black hole itself doesn't enter our luminosity formula, just the growth rate.

But photons radiating from the inner accretion zone carry momentum and exert pressure. The pressure pushes back on the accreting gas.

There is a maximum luminosity above which outward pressure exceeds the inward pull of gravity, in which case the black hole cannot accrete.

This is called the Eddington luminosity.

A black hole that is twice as massive exerts twice as much gravitational force, so its luminosity limit is twice as high:

$$L_{\mathrm{Edd}} = 3 \times 10^4 L_\odot \left(\frac{M}{M_\odot} \right) ,$$

where M is the black hole mass.

Basic Cosmology

In 1921, Edwin Hubble demonstrated that the "spiral nebulae" seen in telescopes are in fact distant galaxies, like the Milky Way.

Galaxies range widely in size, but a "typical" galaxy is roughly 100,000 light years across and contains 10-100 billion stars.

Over the next decade, Hubble's colleague Milton Humason showed that the spectral lines of most galaxies are shifted to the red, which suggests that they are moving away.

Hubble (1929): A galaxy's redshift is proportional to its distance (twice as far away $\Longrightarrow$ receding twice as fast). The universe is expanding.

This discovery, combined with theoretical calculations based on General Relativity, led to The Big Bang Theory: The universe has expanded from a very hot, very dense state, which existed at some finite time in the past (about 14 billion years ago).

Key points for our purposes:

- Stars are point sources of light, and their spectra show only small redshifts or blueshifts (up to a few hundred $\mathrm{km\,s^{-1}}$).
- Galaxies are extended sources of light, and their spectra are (nearly) aways redshifted, often by thousands or tens of thousands of $\mathrm{km\,s^{-1}}$.
- The higher the redshift the more distant the galaxy.

The Discovery of Quasars

Early radio telescopes detected many sources over the sky.

Initially hard to identify with optical sources because of poor resolution of radio telescopes.

Resolution gradually improved by using linked networks of radio telescopes.

Many sources appeared to coincide with galaxies.

Better resolution showed emission often coming from "lobes" on either side of galaxy, hundreds of thousands of light years from the stars.

A few sources appeared to coincide with stars.

Weird, "radio stars" not previously known.

Astronomers obtained visible light spectra for some of these sources.

Spectra showed lines at completely unknown wavelengths.

Maarten Schmidt (1963): Lines in the source 3C 273 are from hydrogen, but redshifted by 48,000 $\mathrm{km\,s^{-1}}$, 16% of the speed of light!

Lines in the source 3C 48 are from other kinds of atoms, redshifted by 37% of the speed of light.

Implication: These "points of light" are enormously distant and therefore enormously luminous, 100-1000 times more luminous than an entire galaxy of stars.

Named "quasars" for quasi-stellar radio sources, though we now also know many similar objects with little or no radio emission.

Quasar Energetics

Observed quasars have luminosities up to $5 \times 10^{13} L_{\odot}$ (or even more).

(The total luminosity of all stars in Milky Way is about $3 \times 10^{10} L_{\odot}$; the brightest quasars are more than 1000 times more luminous.)

Quasars vary substantially in brightness on timescales of days $\Longrightarrow$ *emission region must be at most a few light days across.*

Some quasars are associated with radio lobes and radio jets. The total energy in the lobes is around 10^{54} joules, about $10^7 M_{\odot} c^2$.

Accretion onto black holes is the only known energy source efficient enough to produce so much energy in such a small volume. The factor of $\frac{1}{12} \approx 0.1$ in the accretion formula means that an accretion disk is about ten times more efficient than fusion of hydrogen into helium (which converts only 0.01 of the hydrogen mass into energy).

The black holes must be massive or they could not emit so much energy without exceeding the Eddington luminosity limit.

The brightest quasars have luminosities equal to the Eddington limit for $M = 10^9 M_{\odot}$, or even a little higher.

Unlike typical stars, quasars emit their radiation over a wide range of photon energies: X-rays, UV light, visible light, infrared light; some are bright in radio waves and in γ-rays.

The properties of the emission are generally what we expect for accretion disks around supermassive black holes.

Especially significant: X-ray iron lines, which arise in hot gas fairly close to the event horizon, show large Doppler shifts and gravitational redshifts.

Also, the radio jets (and sometimes visible light or X-ray jets) that we see in some quasars could be produced by material ejected close to the speed of light, perpendicular to the accretion disk.

Jets can stay straight for millions of years because of the "gyroscope" provided by the spinning BH.

Quasar Demographics

Finding quasars:

- With visible light images alone, look for "stars" (points of light) with very weird colors.
- Or look for visible-light "stars" that have counterparts in X-rays or radio.
- To confirm that an object is a quasar, and measure its distance (from redshift), have to measure its spectrum.

There are now close to a million quasars known.

The most distant known quasars have light redshifted by a factor of 11 $\Longrightarrow$ light emitted when universe was about 0.5 billion years old (compared to 14 billion years today).

Bright quasars were most common 10 billion years ago, much less common today.

The black holes themselves don't go away, so it must be that they are "fed" less often at later times.

Quasars and galaxies

Sharp imaging from *Hubble Space Telescope*, some ground-based telescopes $\Longrightarrow$ quasars live at the centers of the galaxies.

Sharp images needed because quasar can be much brighter than host galaxy itself.

Hosts often appear messed up, suggesting mergers of galaxies may trigger galactic scale gas flows that feed quasars.

Brighter quasars generally reside in bigger host galaxies.

Some galaxies (about 1-2%) have bright central nuclei, which appear to be scaled down versions of quasars, less luminous because black hole is less massive or because a massive black hole is being fed slowly.

We also find evidence for "dormant" black holes at the center of massive galaxies.

Their masses can be estimated from the motions of stars and gas near the center of the galaxy, using our formula

$$M \approx \frac{v^2 r}{G}.$$

The equation is approximate because stars are not actually on circular orbits.

Doing this requires measuring near the center of galaxies, so that the gravity of the black hole dominates over the gravity of the stars.

The case for dormant black holes became much more convincing with *Hubble Space Telescope* because sharp imaging made it possible to measure much closer to galaxy centers.

Current evidence suggests that *all* galaxies with central bulges of stars also have central supermassive black holes.

The BH mass and bulge mass appear to be tightly correlated, i.e., galaxies with more massive bulges have more massive central BHs.

Specifically, the mass of the black hole is typically about 1/1000 of the mass of stars in the bulge.

As we have discussed the motions of stars near the center of the Milky Way imply that our galaxy has a central black hole with a mass of $4 \times 10^6 M_\odot$.

The total luminosity (mostly in infrared and radio emission) is not much larger than $L_\odot$, *very* far below Eddington limit.

It must be getting very thin dribble of gas (but not zero).

Black Holes and Galaxy Formation

Quasars live in galaxies, appear to be associated with "big growth events" in galaxy life.

Omnipresent central black holes in local galaxies $\Longrightarrow$ most galaxies had a "quasar phase."

Quasars are much rarer than galaxies $\Longrightarrow$ quasar phase only lasts a small fraction of galaxy's life.

Close correlation between black hole mass and bulge stellar mass $\Longrightarrow$ one affects the other, but we don't know which way.

Option 1: Gravity provided by stars, or mass lost from stars, determines how big the BH can grow.

Option 2: When BH gets big enough, it drives gas out of galaxy, truncating further star formation.

Influence of black holes on galaxies and *vice versa* is a big open issue in the theory of galaxy formation and quasar evolution.

Summary of Empirical Evidence for Supermassive BHs

- High luminosity, rapid variability of quasars $\Longrightarrow$ enormous energy produced in a small volume. Accretion onto BH is only known mechanism to achieve this.
- Spectrum of radiation has properties expected for BH accretion.
- X-ray iron lines show large Doppler shifts and gravitational redshifts.
- Jets powering radio galaxies could come from BHs. No other ideas how to create such energetic jets or keep them so straight.
- Motions of stars at Galactic Center show large dark mass in small volume.
- Motions of stars and gas near centers of other galaxies show large dark mass in central regions.
- *New* (and discussed more later): Image of accretion flow and shadow cast by the event horizon from the $10^9 M_\odot$ BH in the galaxy Messier 87.