11. Gravitational Waves

Gravitational Waves

General Relativity predicts the existence of gravitational waves, ripples of spacetime curvature that propagate at c.

Analogous to waves on a rubber sheet, or vibrations in the "grid" of spacetime.

If a gravitational wave passes between two observers, the spatial separation between them oscillates.

The wave is characterized by a direction, a period of oscillation, and an amplitude called the *strain* and denoted h:

$$h = \frac{\Delta d}{d} = \text{fractional change of distance},$$

where Δd is the amount of oscillation produced for observers separated by d.

The *energy* of the wave is $\propto h^2$.

Einstein already discussed gravitational waves in a 1916 article describing GR, and he calculated the rate at which they could be produced by accelerating masses in a 1918 article.

But detecting them is hard!

Sources of Gravitational Waves

Gravitational waves are produced by accelerating masses.

Most likely strong sources:

- Binary black holes
- Binary neutron stars (or NS-BH binaries)
- Supernova explosions

Widely separated binaries (many $R_{\rm Sch}$ separation) are steady sources, period of wave is orbital period.

As gravity waves carry off energy, orbital separation of binary components shrinks, eventually leading to a merger (see Thorne Figs. 10.1, 10.2, 10.9).

Waves are strongest when sources are close to merging: strong curvature rapidly changing.

A merger produces a short burst of strong gravity waves at the end of a long steady stream of weaker gravity waves.

For *merging* black holes at distance D, the strain is roughly

$$h \approx \frac{1}{5}\frac{R_{\rm Sch}}{D},$$

where $R_{\rm Sch}$ is the Schwarzschild radius of the black holes.

Falls with distance as the wave spreads out.

The characteristic period of the waves from a merger is similar to the orbital period when the black hole event horizons are “touching”:

$$P \approx \frac{2\pi R_{\rm Sch}}{c} \approx 10^{-3}\left(\frac{M}{10\,M_\odot}\right)\,{\rm s}.$$

(This is a very approximate formula, which somewhat underestimates the period.)

Indirect Discovery of Gravitational Gaves: The Binary Pulsar

Strong empirical evidence for gravity waves comes from observations of the Hulse-Taylor binary pulsar, discovered in 1975.

These are two neutron stars (one of which we observe as a pulsar), orbiting each other with an eight-hour period.

The orbit can be measured very precisely by timing the arrival of the radio pulses.

This binary system should be emitting gravitational waves and losing energy.

The binary orbit loses energy and shrinks at exactly the rate predicted by GR, to within 1%.

The orbital period changes by 77 microseconds (77 millionths of a second) per year, implying that the two neutron stars will spiral together and merge about 300 million years from now.

Russell Hulse and Joe Taylor won the Nobel Prize for this discovery work in 1993.

Direct Detection: What Sensitivity is Needed?

We’d like to detect gravity waves directly and use them as a new tool for observing the universe.

Before the first direct detections, you needed to make a guess about the sensitivity needed to do this in order to design an experiment. Here is a very rough guess.

Suppose a black hole merger occurs once per million years per galaxy. (This million year number is the most uncertain part of the calculation. We know that there is about one supernova per 100 years per galaxy, and we know that Hulse and Taylor discovered one binary neutron star in our galaxy that will merge in 300 million years.)

To build a detector that will detect one event per year, we should be sensitive to a volume that contains a million galaxies.

This requires going out to a distance of about 2 billion light years. (Based on the average density of galaxies in space; take my word for it.)

Consider a merging $10M_\odot$ BH binary at a distance of 2 billion light years, the predicted strain is

$$h \approx \frac{1}{5} \times \frac{30\ {\rm km}}{(3\times 10^5\,{\rm km\,s^{-1}})(3\times 10^7{\rm s/yr})(2\times 10^9{\rm yr})} \approx \frac{1}{30}\times 10^{-20} \approx 3\times 10^{-22}\ .$$

The design and construction of LIGO was based on an argument roughly like this.

Thanks to LIGO's detections, we now know that the black hole merger rate is roughly 30 per million years per galaxy, making things slightly easier.

LIGO

The first gravitational wave detectors were finely tuned metal bars, which could be vibrated by a passing wave and ring like a (very quiet) bell.

The best sensitivity achieved by these bar detectors is about 10^{-17}, so thousands of times too low if the estimate above is correct.

The Laser Interferometer Gravitational-Wave Observatory (LIGO) uses a different technique, measuring distances with a laser interferometer.

It uses two L-shaped laser interferometers (in Louisiana and Washington), with each arm 4 km long.

A passing gravity wave would change the length of one arm relative to another, and this change is detected by the interference pattern of the laser light with itself.

For $h \sim 3 \times 10^{-22}$, the change in arm length is 4 km $\times(3 \times 10^{-22}) \sim 10^{-18}$ m, roughly 1/1000 of the diameter of an atomic nucleus.

This is approximately the sensitivity of LIGO!

Challenges: Need seismic isolation, powerful lasers, perfect optics, excellent vacuum system.

LIGO is sensitive to gravitational waves with periods of about 0.0001 sec to 0.1 sec.

LIGO is thus suited to detecting gravitational waves from merging stellar mass black holes or neutron stars, which have characteristic periods $\sim 10^{-3}$s.

If a supernova occurs in our own Galaxy, the formation of the neutron star might create gravitational waves strong enough to detect.

Direct Discovery of Gravitational Waves

After more than 20 years of design, development, construction, and steady improvements to its technology, LIGO began an observing run in September 2015 and within two days (9/14/2015) detected a remarkably clear signal in both of the detectors (Washington and Louisiana).

The detectable signal lasts about 0.2 seconds, and during that time the strength of oscillations increases and the period decreases, before cutting off abruptly.

The observed signal is a good match to GR predictions for merging black holes.

The peak strain is $h \approx 10^{-21}$, and the period of the last oscillation is about 0.01 sec.

LIGO inferred a mass of $30M_\odot$ for these black holes and a distance of just over 1 billion light years.

LIGO is a collaboration of about 1000 scientists, most but not all in the U.S.

In 2017, Kip Thorne, Rainer Weiss, and Barry Barish were awarded the Nobel Prize for their central roles in creating LIGO and making it a success.

LIGO Discoveries So Far

As of early 2025, LIGO has detected roughly 90 gravitational wave events.

Most of these are mergers of two black holes, which produce the strongest signals and can thus be seen out to the largest distances.

There are also several detected mergers of a black hole with a neutron star, and at least one (maybe several) mergers of two neutron stars.

An individual LIGO detection gives:

- Location on the sky, but with rather low precision (like early radio telescopes).
- Mass of the two black holes (from the period of oscillation).
- Distance (from the strength of the signal, knowing the mass).
- Spins of the two black holes from the detailed structure of the waveform, though these are not measured very precisely in most cases.

We have learned that black holes of $30 - 60 M_\odot$ exist and merge more often than we might have guessed.

Nearly all black holes discovered by X-ray observations and binary companions prior to LIGO had masses below $20 M_\odot$.

Most importantly, we have learned that black hole mergers produce gravitational waves with properties very close to GR predictions, an astounding success of Einstein's theory of gravity.

The merger of two neutron stars detected by LIGO in 2017 was also detected in gamma rays and visible light.

Visible light observations showed that this neutron star merger produced lots of heavy elements like gold and platinum. Neutron star mergers, even though they are a thousand times less common than supernovae, may be the main source of these rare elements.

LISA

To get to longer periods, we need to go to space.

The Laser Interferometer Space Antenna is a planned European space mission, with NASA participation, that would use 3 satellites in solar orbit, separated by 2.5 million km.

Laser interferometers would measure the separations of the satellites with a precision of 10 trillionths of a meter (0.01 nano-meters), providing sensitivity to a strain

$$h \sim \frac{10^{-11}\,\mathrm{m}}{5 \times 10^{9}\,\mathrm{m}} \sim 10^{-21}.$$

LISA is currently scheduled for launch in 2035, set partly by the time needed to develop and test the technology and build the mission, and partly by the availability of funding.

Because of larger separation and lack of seismic interference, LISA is sensitive to longer period waves, roughly 10 sec - 10^4 sec.

For merging black holes, the minimum period corresponds to $M \sim 10^6 M_\odot$.

The sensitivity of LISA is enough to measure black hole mergers in this mass range with high precision out to the farthest reaches of the observable universe, 10 billion light years away.

LISA could also measure *steady* gravity waves from stellar mass binaries (black holes, neutron stars, white dwarfs) with periods of ~ 1000 seconds.

One of the most interesting sources for LISA will be stellar mass black holes that spiral in (because of gravitational wave emission) to a central supermassive black hole, eventually merging.

Warping of spacetime by the spin of the central black hole makes the inspiral orbit very intricate (see video on course web page).

LISA can detect gravitational waves from such an inspiral over many orbits, allowing highly detailed tests of GR predictions.

Pulsar Timing Arrays

We skipped this topic because of limited time. I leave the notes here for reference, but we will not cover this topic on the final exam.

Rapidly spinning pulsars are extremely stable clocks.

To detect gravitational waves with very long periods, of months to decades, one can monitor the light-travel times to pulsars all over the sky.

This is hard, but over the last few years a big collaboration using multiple radio telescopes has presented solid evidence for a background of long-period gravitational waves, detected through pulsar timing.

Long period waves would come from orbiting supermassive black hole binaries, long before they merge.

This experiment doesn't isolate individual sources – at least not yet – just the overall background.

The level of the background is about 2-3 times higher than predicted based on what we know about the population of supermassive black holes, which is an interesting puzzle.

Prospects and Questions for Gravitational Wave Astronomy

Ground-based interferometers will continue to improve in sensitivity.

Higher sensitivity will

- Increase number of events (by detecting them to greater distances)
- Allow detection of "quieter" events that produce intrinsically weaker signals
- More precisely measure the waveform signals: Better measurements of interesting quantities like black hole spin. More stringent tests of whether the signals agree with GR predictions.

More interferometers located around the world will improve localization, making it easier to search for electromagnetic signals coming from the events that produce the graviational waves.

Three big open questions for gravitational wave astronomy:

- What gravitational wave events also produce detectable signals in electromagnetic radiation (gamma rays, X-rays, visible light, radio waves)?
- Are there kinds of sources we haven't anticipated? (Weird classes of stellar explosions. Signals from extreme processes in the very early universe.)
- Are there deviations from General Relativity that could show that we need to revise our theory of gravity?