13. Black Hole Evaporation

Quantum Mechanics

The *other* great revolution (besides relativity) of 20th century physics (mostly 1900-1927).

Key ideas:

Discreteness: Energy comes in discrete units (not continuous). For example, electron energy levels in atoms are discrete, vs. planetary orbits in the solar system which we normally describe as continuous. Photons are discrete packets of energy, while the wave description of EM radiation is continuous.

Uncertainty:

- Some experimental outcomes can *only* be predicted probabilistically. (E.g., probability of a radioactive decay in some amount of time.)
- Position and momentum are "complementary" quantities that cannot both be well determined at the same time. (This underlies degeneracy pressure.)

"Classical" physics describes phenomena as continuous and deterministic.

The effects of quantum mechanics are mainly evident on sub-atomic scales.

On macroscopic scales, small spacing of energy levels and averaging of many events makes "classical" descriptions (continuous, deterministic) very accurate.

As formulated by Einstein, Special Relativity and General Relativity are "classical" theories.

There is a successful (i.e., physically consistent and experimentally well tested) combination of quantum mechanics and *special* relativity called quantum field theory (developed in 1930s, 1940s, and 1950s).

It is the basis of our modern understanding of sub-atomic particles and the interactions that govern them.

Thermodynamics

One of the great revolutions (along with electromagnetism) of 19th century physics.

Thermodynamics describes the physics of energy, heat, temperature, pressure, for systems composed of many atoms (gases, fluids).

Two basic "laws" (discovered empirically):

1. Energy is always conserved (though it may be transformed).

2. The entropy of a closed system can increase, but it can never decrease. (One consequence: a hot body can heat up a cold body, but never *vice versa*.)

For #1 to hold in general, the "bookkeeping" must include gravitational potential energy and $E = mc^2$ energy.

Entropy can be expressed in terms of temperature and density, but we now understand the second law (and many other aspects of thermodynamics) as consequences of the statistical behavior of large numbers of particles.

Entropy is a quantitative measure of disorder, the logarithm of the number of ways a system could be rearranged without changing its large scale properties.

For a system made of many, many particles, it is hugely more probable for disorder to increase rather than decrease.

The 2nd law refers to *total* entropy — e.g., we can get increasing order on earth because we're getting energy from the sun (which is creating entropy by radiating photons into space).

Black Hole Thermodynamics

1970: Stephen Hawking shows that in mergers of black holes, the total area of event horizons must always increase.

He and others (especially Jacob Bekenstein) note *very* close analogies between equations governing black holes and equations of thermodynamics.

Horizon area is like entropy, always increases.

Could explain why dropping stuff (with entropy) into a black hole doesn't violate the 2nd law of thermodynamics: the black hole horizon area increases, hence the entropy increases.

The Problem:

In these analogies, the black hole "surface gravity" (roughly $GM/R_{\rm Sch}^2$) plays the role of temperature.

In thermodynamics, any body hotter than its surroundings radiates energy.

Black holes don't radiate.

Hawking Radiation

Another consequence of quantum uncertainty: over short time periods, energy fluctuates.

"Empty space," a.k.a. "the vacuum," is far from empty. It is filled with "virtual particles," which pop into existence in pairs, then disappear soon after.

Despite their ghostly existence, these virtual particles have real effects that are measured experimentally to high precision.

1974: Hawking investigates behavior of virtual particles near event horizons.

Consider a pair of virtual photons with wavelength $\lambda \sim R_{\rm sch}$, created near the event horizon.

Tidal gravity of the black hole pulls them apart, gives enough energy to make them real, long-lived photons.

One falls into the event horizon, but one can escape.

Thus, "radiation" comes from the black hole.

The energy of the escaping photon must come from the black hole, so its mass goes down.

Implication: the laws of black hole thermodynamics really do describe the entropy and temperature of black holes, which are a consequence of quantum mechanics in the presence of strong gravity.

A black hole's entropy is the logarithm of the number of ways that the black hole could have been made.

Black Hole Evaporation

If a black hole is left on its own for long enough, it will radiate away all of its mass and disappear. *BUT*

For a stellar mass black hole, the temperature is *extremely* low, e.g., 3×10^{-8} degrees above absolute zero for $2M_{\odot}$.

The lifetime is 10^{67} years compared to the age of the universe, which is 10^{10} years.

Even an isolated black hole is growing by accreting background photons at least a billion times faster than it is shrinking by Hawking radiation.

But if the universe lives forever and keeps getting colder, all the black holes will eventually evaporate.

IF mountain-mass black holes ($M \sim 10^{12}$ kg) formed in the very early universe, then they would be evaporating today, in explosions of gamma-rays.

This would be cool, but there is no evidence that these evaporating black holes exist, and no reason to think that they ought to.

A rough calculation of black hole evaporation

(I will use $\sim$ in place of $=$ to indicate that these are only approximate equations.)

The characteristic length associated with a black hole of mass M is the Schwarzschild radius $R_{\text{Sch}} = 2GM/c^2$.

What time can we associate with this length? The light-crossing time $t_c = R_{\text{Sch}}/c$.

If Hawking radiation exists at all, we expect the black hole to emit roughly one photon of wavelength $\lambda \sim R_{\text{Sch}}$ per light-crossing time t_c, because there are no other numbers around to set length and time scales.

A photon of wavelength λ has energy $E_\gamma = hc/\lambda$, where h is Planck's constant.

The Hawking luminosity of the BH — the rate at which it radiates energy — is

$$L \sim \frac{(hc/\lambda)}{t_c} \sim \frac{hc^2}{R_{\text{Sch}}^2} \ .$$

The total energy available to be radiated is Mc^2. The time for the BH to evaporate is the available energy divided by the rate at which energy is being radiated,

$$t_{\text{evap}} = \frac{Mc^2}{L} = \frac{MR_{\text{Sch}}^2}{h} \ .$$

After substituting numbers one finds

$$t_{\text{evap}} \approx 2 \times 10^{64}\,\text{yr} \left(\frac{M}{3M_{\odot}}\right)^3 \ .$$

The 10^{67} yr number given above is more accurate, because this is a rather crude calculation, but the concepts are right.

Hawking Radiation: Key Takeaways

Motivated by black hole thermodynamics, especially the analogy between horizon area and entropy.

Calculated (by Hawking) by applying quantum mechanics to strongly curved spacetime near event horizons.

For stellar mass or supermassive black holes, Hawking radiation is extremely work. It is *not* the source of energy that we see coming from X-ray binaries or quasars.

While the existence of Hawking radiation is *not* empirically confirmed, it must exist if quantum mechanics and General Relativity are both correct.

It demonstrates an important point of principle: given enough time, black holes evaporate.

The fact that the analogy between the classical GR theory of black holes and the classical theory of thermodynamics was ultimately vindicated by the physics of quantum fields in curved spacetime is an almost spooky example of the power of equations in natural science.

Quantum Gravity and General Relativity

Hawking and others developed the theory of *quantum fields in curved spacetime.*

This theory *involves* quantum mechanics and GR, but it does not fully merge them.

One of the highest ambitions and toughest challenges of fundamental physics is to develop a theory of *quantum gravity*, in which spacetime curvature itself is treated quantum mechanically.

Hawking radiation is a useful clue to developing this theory.

If we ever develop a successful theory of quantum gravity, it should reduce to General Relativity in the limit of macroscopic scales and finite curvature.

Analogous to the way that GR reduces to Newtonian gravity in the limit of slow speeds and weak curvature, or to Special Relativity in the case of flat spacetime.

There is a natural (very tiny) scale where we expect that quantum gravity *must* become important, known as the Planck length, $l_p = 10^{-33}\,\mathrm{cm}$.

(Very roughly, the ratio of the Planck length to the size of an atom is like the ratio of the size of an atom to the size of the solar system.)

We do not know whether quantum gravity will have a very different conceptual basis, like the difference between GR and Newtonian gravity.

We do not know whether there are observable effects of quantum gravity on scales that we can probe with feasible experiments or astronomical observations.

String theory is the best candidate for something that might become a successful theory of quantum gravity.

In string theory, fundamental "particles" are really tiny loops of string oscillating in a 10-dimensional space, so the conceptual change is fairly radical.

Calculating observable quantities in string theory is very difficult, so even though the ideas have been around for four decades, we don't have empirical evidence on whether string theory is correct or incorrect.