5. Special Relativity

This material is covered in Chapter 1 of Thorne's book.

If you want to know more, there are many treatments of special relativity either at a popular level or at levels similar to or moderately higher than we will cover in class. These include sections in high school or college physics texts, popular books, and web sites. Two notable ones are *Spacetime Physics* by Edwin Taylor and John Archibald Wheeler and *Relativity: The Special and General Theory – A Clear Explanation that Anyone Can Understand*, by Albert Einstein. The level of the first one is somewhat higher than this course, and the level of the second is about equal to that of this course.

Space and Time

What is time? How do we know time is passing?

Stuff happens.

Merriam-Webster: Def. 1b – a nonspatial continuum that is measured in terms of events which succeed one another from the past through present to future.

Wikipedia: Time is a basic component of the measuring system used to sequence events, to compare the durations of events and the intervals between them, and to quantify the motions of objects.

What is space?

Merriam-Webster: Def. 4a – a boundless three-dimensional extent in which objects and events occur and have relative position and direction. Def. 4b – physical space independent of what occupies it (absolute space).

Events happen in space and time — i.e., we can assign a location and a time to any event.

At least, that is our everyday intution.

Puzzles of Light and Reference Frames

I throw a ball at 50 m.p.h. on a train moving 50 m.p.h. How fast does the ball go?

I shine a flashlight on a train moving at $0.5c$. How fast does the light go?

Maxwell's equations predict constant speed of light.

With respect to what? Luminiferous ether?

Various attempts to measure motion with respect to ether, all unsuccessful. The most famous (discussed in the book) is the Michelson-Morley experiment.

Would establish preferred reference frame.

Contrary to Galilean/Newtonian physics, where only relative motions enter.

Motivation and approach

Einstein 1905: *On the Electrodynamics of Moving Bodies*

Motivations:

- Newtonian mechanics: Only relative motions enter, no preferred "absolute" reference frame

- Fixed speed of light seems to violate this, but no experiment shows such violation
- Also, electromagnetic phenomena in different frames have different descriptions, same effect

Structure of paper: Impose two postulates, work out consequences

Postulates:

1. All reference frames in uniform relative motion are equivalent; there is no absolute rest frame.
2. The speed of light is a universal constant c, independent of the speed of the emitting body.

Arguably 2 could be folded into 1.

The restriction to uniform (constant speed and direction) frames, motivated by Newtonian mechanics, is the reason this is called "special" relativity. Will be relaxed in general relativity.

Postulate 2 implies: shine a flashlight on a moving train, the light still propagates at c. Seems impossible at first glance, but this is because our intuition uses incorrect concepts of space and time.

Relativity of simultaneity

Warmup: You're watching a rock concert from the far end of Ohio Stadium. You see something explode on stage, and 0.5 seconds later you hear a boom. What happened first, the flash or the boom?

FIGURE 1: TRAIN CAR

Train example:

Charles sees Alice and Bill get struck by lightning, just as Carol passes him.

Who got struck first?

According to Charles: They happened at equal distance, seen at same time $\longrightarrow$ events were simultaneous.

According to Carol: Sees Alice get struck first. Same distance away $\longrightarrow$ Alice really did get struck first.

Who is right?

Both of them.

Relativity of simultaneity: Events that appear simultaneous to an observer C do not appear simultaneous to an observer C' moving relative to C.

Implication: There is no absolute significance of simultaneity, hence no absolute time.

Relativity of length
Charles is actually in a tunnel. When Carol passes him, he shuts doors at two ends of tunnel, precisely enclosing the train.

Carol sees forward door of tunnel close first. Back door hasn't closed yet, train is still sticking out of tunnel.

How is this possible?
Charles thinks train is shorter than Carol does.
Who is right?
Both of them.

Relativity of length: If two observers are moving relative to each other, they measure different lengths and distances. A moving object is shortened along its direction of motion.

Since, precisely speaking, distances must be measured between simultaneous events, we should not be too surprised that relativity of simultaneity leads to relativity of length.

Time dilation
In order to quantify this, we need a good clock.
If c is universal, a mirrored light box is a perfect clock.
For a light clock $l = 1\text{m}$ tall, how long does it take for light to go bottom to top?
$t = l/c = 1\text{m}/(3 \times 10^8\text{m/s}) = 3.33 \times 10^{-9}\text{s}$

FIGURE 2: WOMAN ON SKATEBOARD

Woman on skateboard, moving to right at speed v.
According to her, how long does it take to get from A to C? From A to B?
What does stationary observer (man) think? Light has gone further, so more time must be required.
Woman thinks time from A to B is $t = l/c$, man thinks time is $t' > t$.

According to man, what is horizontal distance from A to B? vt'.
What is distance x light traveled between A and B? $x = \sqrt{l^2 + (vt')^2}$, by Pythagorean theorem.
What is relation between x, c, and t'? $x = ct'$.
Substitute:

$$\begin{aligned} ct' &= \sqrt{c^2t^2 + v^2t'^2} \\ c^2t'^2 &= c^2t^2 + v^2t'^2 \\ t'^2(c^2 - v^2) &= c^2t^2 \\ t'^2\left(1 - \frac{v^2}{c^2}\right) &= t^2 \\ t' &= \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}. \end{aligned}$$

In relativity we often refer to the *Lorentz factor*

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} > 1. \tag{5.1}$$

($\gamma = 1$ if $v = 0$.)

The man thinks the woman's clock "runs slow" by a factor of γ.
What she thinks takes time t, he thinks takes time

$$t' = \gamma t > t\ . \tag{5.2}$$

$t =$ time recorded in moving frame
$t' =$ time recorded in stationary frame
True for any clock: electromagnetic, mechanical, biological, etc.

What does the woman think about the man's clock?
By symmetry, same thing: it *also* runs slow.

Why don't we notice this?
For $v = 100$ km/hour, $t' = 1.0000000000000044\, t$. (14 zeroes).
Not noticeable at everyday speeds.
For $v = \frac{4}{5}c = 240,000$ km/s, $t' = \frac{5}{3}t$.

Time dilation: Moving clocks run slow by a factor of γ.

Note that the γ formula doesn't make sense if $v > c$. This suggests we should interpret c as a maximum velocity, a point we will return to later.

Length contraction
There are chalk marks at A, B, and C.
What is the distance from mark A to mark B?
According to him: $d' = vt'$.
According to her: $d = vt$.
She sees distance as shorter by factor of $t/t' = \gamma$.
By symmetry, he sees her and her skateboard as being foreshortened by the same factor.

$$l' = \frac{l}{\gamma} < l\ . \tag{5.3}$$

$l =$ length measured in moving frame
$l' =$ length measured in stationary frame

Length contraction: Moving objects contract in their direction of motion by a factor of γ. We could describe this phenomenon more precisely in terms of spatial separation of simultaneous events in a moving reference frame.

Mr. Tompkins in Wonderland.

Muons

Muons are unstable subatomic particles, with an average lifetime of $\approx 10^{-6}$ seconds.

They can be produced in the laboratory by colliding sub-atomic particles.

They are also produced by cosmic rays colliding with atomic nuclei in the earth's atmosphere, at altitude $\sim 10,000$m.

Traveling at $0.998c$, how far can they get in 10^{-6}s? 300m.

Many actually get to surface of earth, where they can be detected by various means.

How is this possible?

Our point of view: Muon clocks run slow, by γ (≈ 160 for $v = 0.998c$). They live longer (in our frame) because of their high speeds, so they have time to reach the earth's surface.

Muon's point of view: The distance from 10,000m to the ground is shortened by γ, to 62.5 meters. They can make it in 10^{-6}s.

Addition of velocities

The relativity of times and lengths means that velocities do not add in the naive way.

Relativistic velocity combination:

One can derive a formula for the combination of velocities to show that an object thrown at speed w on, e.g., a train car moving at speed v, has a velocity that

- is almost exactly $v + w$ if v and w are both much smaller than c
- is always less than $v + w$
- is always less than c if v and w are less than c
- is equal to c if v or w equals c

The formula (which you don't need to remember but is interesting to see) is:

$$u = \frac{v + w}{1 + (vw/c^2)}.$$

Throwing a baseball at $0.2c$ on a train moving at $0.9c$ does not cause the baseball to move faster than c.

Relativistic mass increase

Relativistic mass increase: If an object has mass m_0 when it is at rest, it has mass γm_0 when it is moving at speed v.

(More precisely, the momentum is $\gamma m_0 v$.)

We could demonstrate this with a thought experiment like the one we used for time dilation and length contraction, but we will skip this so that we can get on to other subjects.

Mass increase allows us to understand why a material object cannot go faster than c.

As its speed approaches c, its mass increases indefinitely, so an infinite amount of force would be required to actually accelerate it to c.

Relativistic Energy

Shortly after the *Electrodynamics of Moving Bodies* paper, Einstein came up with the argument (given earlier) for his famous equation

$$E = mc^2. \tag{5.4}$$

Implication: Mass is a form of energy, and the conversion between mass and energy is given by the large factor c^2.

Any release of energy E is accompanied by a decrease in mass by an amount E/c^2.

While it was at first proposed as a conjecture, this equivalence between mass and energy has been amply confirmed by nuclear fusion as the source of energy in stars and in hydrogen bombs, and by direct experiments.

A photon is never at rest, so it doesn't really make sense to talk about its rest mass. The "inertia" (resistance to acceleration) associated with a photon of energy E is equivalent to a mass of E/c^2.

Special Relativity

Einstein's postulates

- The equivalence of all frames in uniform relative motion
- The constancy of c

imply

- Simultaneity of events is relative (no absolute time)
- Lengths of objects are relative (no absolute space)
- Time dilation: moving clocks run slow
- Length contraction: moving objects compress in direction of motion
- Mass increase: Moving objects have higher inertial mass, harder to accelerate
- $E = mc^2$

The strength of the time dilation, length contraction, and mass increase effects depends on $\gamma = 1/\sqrt{1 - v^2/c^2}$, so they are difficult to measure at velocities $v \ll c$.

Empirical Evidence for Special Relativity

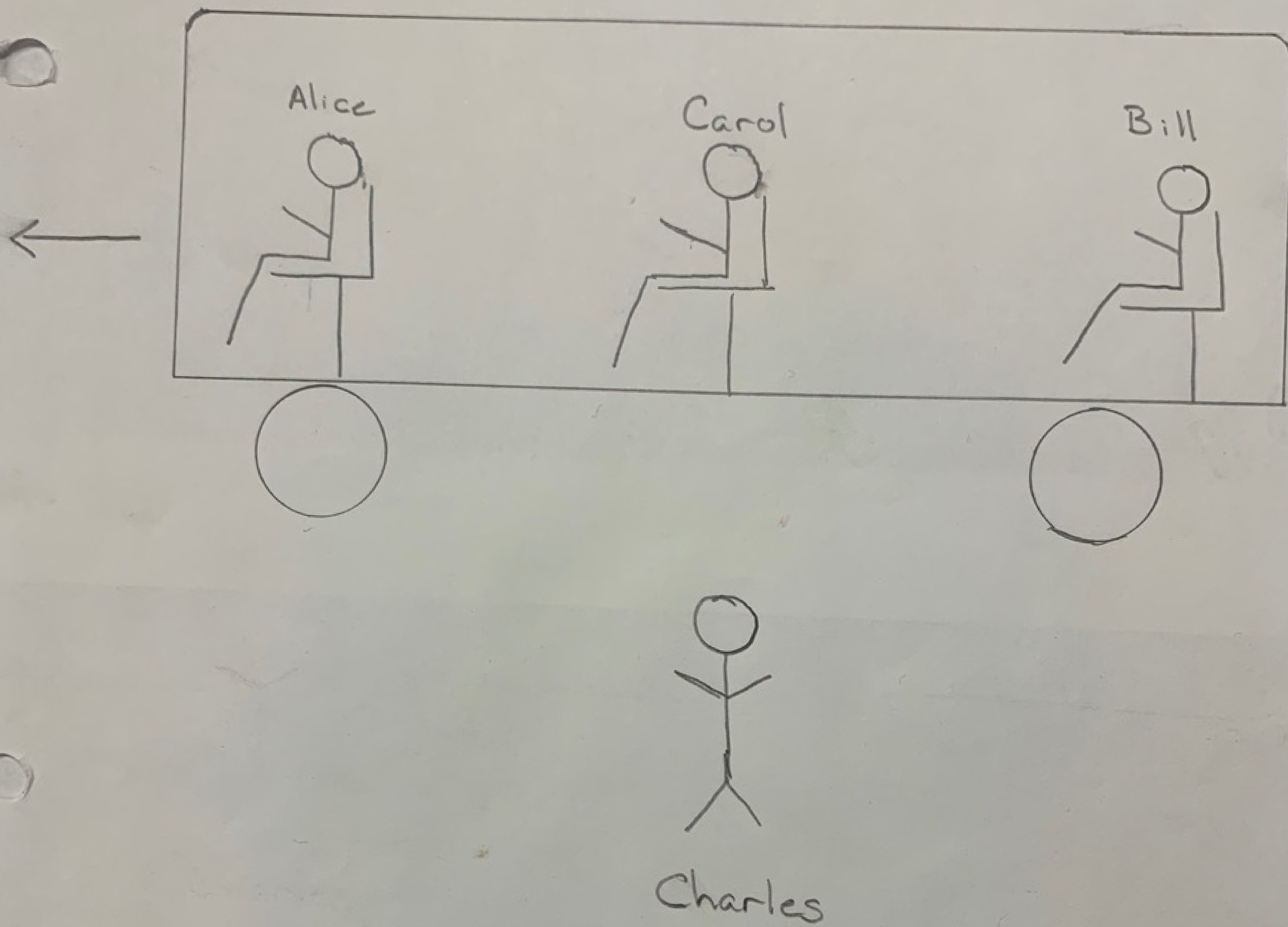
- Constancy of c, independent of earth motion (Michelson-Morley experiment).
- Einstein uses relativity to calculate many effects of electrodynamics, some of them surprising — aberration of starlight, radiation pressure, relativistic Doppler effect — and all of them eventually confirmed by experiment.
- Muons produced in atmosphere reach earth.
- Clocks on airplanes run slow.
- Particle accelerators must apply more force to accelerate particles as they approach c.
- $E = mc^2$: nuclear energy, powering of stars
- Many high precision tests of time dilation, length contraction.

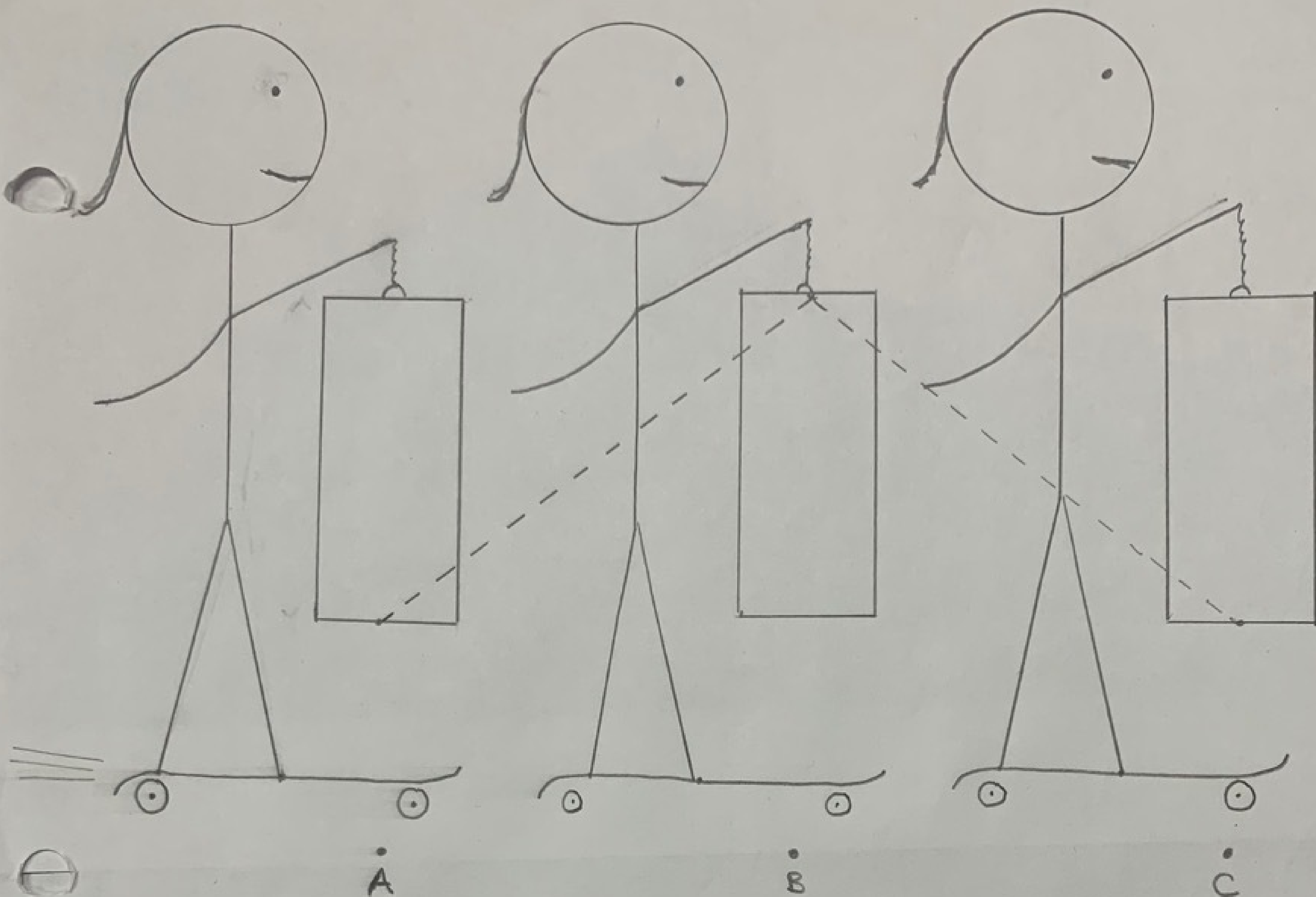
For example, GPS satellite velocities are about 4 km/s, so $\gamma = (1 - v^2/c^2)^{-1/2} \approx 1 + 10^{-10}$.

If one ignored time dilation of clocks, one would get off by 1 nanosecond (10^{-9} sec) every 10 seconds.

The speed of light is about 1 foot per nanosecond, so your GPS positioning would get off by about 1 foot every 10 seconds, or 360 feet per hour.

GPS must also account for the effect of gravity on clocks, but that's a topic for the next section.


Alice
Carol
Bill
Charles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