Astronomy 2142: Assignment 1

This assignment is due at 5 pm on Friday, Jan 31. The preferred submission is electronic, via Carmen, preferably PDF. You may also submit it on paper, either in class or at my office in 4019 McPherson Lab (if my door is closed, slide it under). It's your responsibility to write clearly enough that we can grade your answers. Write your name on the assignment!

Late assignments will be marked down 10 points if turned in before midnight on Jan 31, or 15 points if turned in before 5 pm on Tuesday, Feb 4. No assignments will be accepted after that time.

You may consult with others in the class when you are working on the homework, but you should make a first attempt at everything on your own before talking to others, and you must write up your eventual answers independently.

You are welcome to come to my office hours or TA Wynne Turner's office hours for advice. This will almost certainly be helpful if you are finding the assignment difficult. Please spend some time working on the assignment before you come to office hours so that you know what your questions are. Office hours this week are different from those on the syllabus, so look below!

In-person office hours, 4030 McPherson Laboratory (4th floor, SW corner)
Wednesday, 1/29, 4pm-5:15pm, with David Weinberg
Thursday, 1/30, 11:15am-12:30pm, with Wynne Turner

Virtual office hours, Zoom 827 776 2849, Passcode A2142
Friday, 1/31, 9:15am-10:15am, with David Weinberg

Part I: Short Questions

Answer each question in one or two sentences. Each question is worth 5 points, except #5 which is worth 10 points.

1. In Thorne's prologue, the protagonist (you) wishes to fly in a rocket close to the event horizon of a black hole. You find that you cannot do this for the stellar mass black hole "Hades," but you eventually fly close to the event horizon of the supermassive black hole "Gargantua." Why can't you fly close to the event horizon of Hades?

2. Why is the black disk of Gargantua's event horizon surrounded by a bright ring?

3. Why are X-ray telescopes good for finding black holes?

4. How do we know that the Milky Way galaxy has a 4 million solar mass black hole at its center?

5. You roll two billiard balls towards each other at a speed of 2 m/sec. One of them is a normal billiard ball with a mass of 0.17 kg, but the other is a super-heavy billiard ball (made of lead alloy) with a mass of 3 kg. After they collide, the normal billiard ball bounces backward, reversing its direction. The super-heavy billiard ball continues moving forward, but at a speed slower than 2 m/sec.

Explain this behavior with reference to Newton's 2nd and 3rd laws.

Part II: Acceleration of the moon

Each part of the question is worth 5 points. If your answer is based on an equation, list the equation as well as the numerical result. For parts (c) and (d), consult the diagram below.

The distance from the earth to the moon is 384,000 km, or 3.84×10^8 meters.

(a) What is the circumference of the moon's orbit?

(b) In 100 seconds, how far does the moon travel in its orbit? (Hint: there are 2.36×10^6 seconds in a month.)

(c) If there were no gravity, then during these 100 seconds the moon would go along the straight-line path **AB** in the diagram (which is not drawn to scale). Instead it follows the curved path **AC**. What is the distance from **O** (the center of the earth) to **B**? Give your answer in meters.

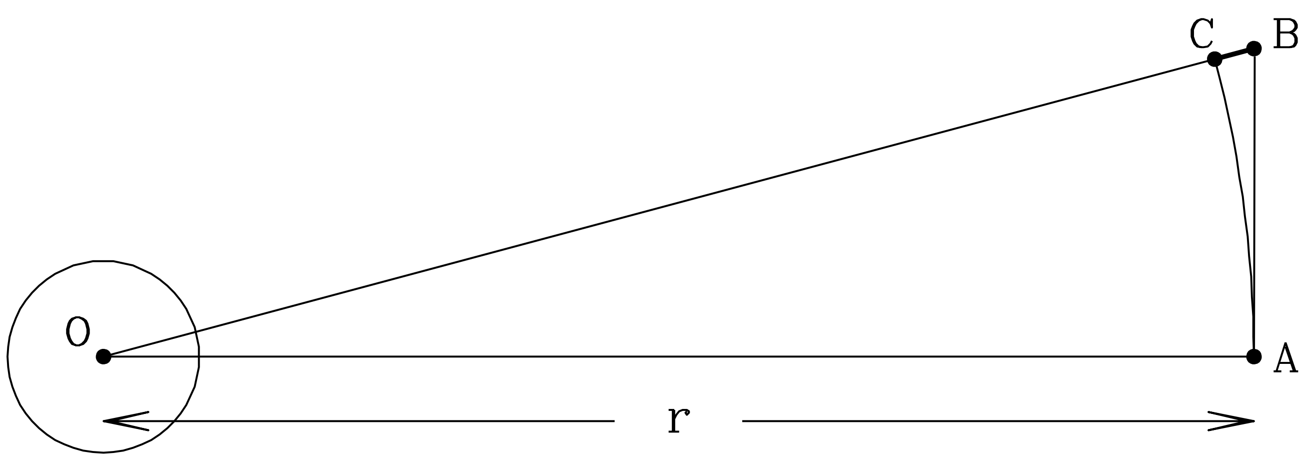
(Hint: Use the Pythagorean theorem, which says that the sides of a right triangle are related by $a^2 = b^2 + c^2$ where a is the longest side.)

(d) How far did the moon "fall" towards the earth by following the curved path **AC** instead of the straight path **AB**? Give your answer in meters.

(Note: If you get zero, it means that you didn't keep enough significant figures when you answered c. It can be difficult to get a calculator to keep the required number of digits, so I will give you a hint — $\sqrt{(3.84 \times 10^8)^2 + (1.02 \times 10^5)^2} = 384,000,013.5$.)

(e) How far would an object dropped from an airplane (flying on earth) fall in 100 seconds, assuming no air resistance?

(f) If all went well, then your answer for (e) is about 3600 times larger than your answer for (d). Why does this factor of 3600 support Isaac Newton's inverse-square law of gravity?



Part III: Understanding and using Kepler's 3rd law

Each part of the question is worth 10 points.

For this problem, you will need to use the equations

$$a = \frac{v^2}{r} \tag{1}$$

for the acceleration of an object moving in a circular path of radius r and

$$a = \frac{GM}{r^2} \tag{2}$$

for the gravitational acceleration produced by a central object of mass M at a distance r. You should also remember that the speed of an object in a circular orbit of radius r and period P is

$$v = \frac{2\pi r}{P}. \tag{3}$$

(a) Combine these equations to show that

$$\frac{GM}{4\pi^2} = \frac{r^3}{P^2}. \tag{4}$$

What is the relation between this equation and Kepler's 3rd law?

("Combine these equations" means write out the algebra steps that take you from equations 1-3 to equation 4. Write words to explain what you are doing if needed.)

(b) The distance from the earth to the sun is called the Astronomical Unit (AU) – i.e., the radius of the earth's orbit is 1 AU. (We won't worry about the slight ellipticity of this orbit.) The radius of Jupiter's orbit is 5.2 AU. Using Kepler's 3rd law, how many years does it take Jupiter to go around the sun?

Show the calculation you did to get your answer (a one-line equation is enough). You can also look up the answer to this question to check that you did it right.

(c) Jupiter's largest moon, Ganymede, orbits Jupiter once every 7.2 days. The distance from Ganymede to Jupiter is 7.15×10^{-3} AU (just over 1 million km). Use this fact, and your knowledge that the earth orbits the sun in 365 days, to show that the sun is about 1000 times more massive than Jupiter. (Your answer should come out between 1000 and 1100.) [Hint: Start by writing equation (4) twice, once with $r_{\rm Earth}$ and $P_{\rm Earth}$ and once with $r_{\rm Gany}$ and $P_{\rm Gany}$. Think carefully about what goes on the left side of the equation in each case.]

(d) You observe an X-ray binary, in which a normal, visible star orbits an optically invisible (but X-ray bright) compact object. You measure the period and radius of the normal star's orbit. How could you use this information to determine the mass of the compact object?