Astronomy 2142: Assignment 2

This assignment is due at 5 pm on Friday, Feb 14. The preferred submission is electronic, via Carmen, preferably PDF. You may also submit it on paper, either in class or at my office in 4019 McPherson Lab (if my door is closed, slide it under). It's your responsibility to write clearly enough that we can grade your answers. Write your name on the assignment!

Late assignments will be marked down 10 points if turned in before midnight on Feb 14, or 15 points if turned in before 5 pm on Tuesday, Feb 18. No assignments will be accepted after that time.

You may consult with others in the class when you are working on the homework, but you should make a first attempt at everything on your own before talking to others, and you must write up your eventual answers independently.

You are welcome to come to my office hours for advice. This will almost certainly be helpful if you are finding the assignment difficult. Please spend some time working on the assignment before you come to office hours so that you know what your questions are. Office hours this week are slightly adjusted from those on the syllabus, so look below.

In-person office hours, 4030 McPherson Laboratory (4th floor, SW corner)
Thursday, 2/6, 11am-12:00pm
Virtual office hours, Zoom 827 776 2849, Passcode A2142
Friday, 1/31, 9:15am-10:15am

You can also ask me questions after class on Monday or Wednesday, and/or you can contact our TA, Wynne Turner (turner.1839@osu.edu) to set up a time to get help.

Part I: Short Questions

Each question is worth 5 points. The last two questions rely on Chapter 2; you will need to read through p. 100 to answer them.

1. How did the discovery of the planet Neptune provide evidence for Newton's theory of universal gravity?

2. You have a box with mirrored interior walls, filled with hot atoms and light. For 30 minutes, you shine a laser into the box through a tiny shutter; then you close the shutter so that the box is perfectly lightproof once again. After you have shined the laser in, is the mass of the box higher than before, lower than before, or the same as before? (Assume you can measure the mass with arbitrary accuracy.)

3. True or False: In the 1880s, Michelson and Morley showed that the earth moves relative to the absolute rest frame defined by the "aether" through which electromagnetic waves propagate.

4. Who said that the concepts of space and time were "doomed to fade away into mere shadows," to be replaced by the concept of four-dimensional spacetime?

5. Since the moon orbits around the earth, it is clear that the moon is substantially affected by the earth's gravity. The moon is 250,000 miles from earth. The space shuttle orbits about 300 miles above the earth's surface, where earth's gravity is much stronger. However, astronauts in the space shuttle are "weightless," and loose objects in the space shuttle float rather than fall to the floor. Why?

Part II: Energy and the Sun

Each part of the question is worth 5 points, except part (f) which is worth 10 points. You may want to refer to the "Transitions" section of the notes (§4), especially the discussion of energy.

The metric unit of energy is the joule. The joule can be expressed in terms of more elementary units:

1 joule = 1 kg m^2 sec^{-2}

Power is a measure of the *rate* at which energy is produced (or consumed). The metric unit of power is the watt, which is equal to one joule/sec (i.e., producing or consume one joule per second).

(a) *What is the kinetic energy of a 1 kg body moving at 1 m/sec?*

(b) *How many joules does a 100-watt light bulb use in ten seconds?*

(c) Sugar has an energy content of 4000 calories per kg (as you can see yourself by looking at nutritional information on a sugar packet). One calorie is the energy required to raise the temperature of one kg of water by one degree centigrade. One calorie is about 4000 joules, so the energy content of sugar is 16×10^6 joules/kg.

Suppose that you "burn" one kg of sugar and use the energy to power a 100-watt light bulb. *How long can you keep the bulb lit?*

(Give your answer in seconds, and convert to hours remembering that there are $60 \times 60 = 3600$ seconds per hour.)

(d) The luminosity of the Sun (the rate at which it radiates energy into space, mostly in the form of visible light) is 4×10^{26} watts. The *mass* of the Sun is 2×10^{30} kg. Suppose that the Sun were a burning ball of sugar. *How long could the Sun last at this luminosity?*

(Convert your answer from seconds to years recalling that there are 3×10^7 seconds in a year. In addition to your final number, clearly write down the numbers that you are multiplying and dividing in order to get it.)

The answer for other chemical fuels, such as gasoline or coal, would be roughly the same.

(e) The age of the oldest rocks on the Earth, estimated from radioactive dating, is 4 billion years. *Why is this a problem for the "burning ball of sugar" theory of the Sun?*

(f) When 1 kg of hydrogen undergoes nuclear fusion, it produces 0.99 kg of helium. (It's 0.993 kg if we're being exact, but we'll approximate this as 0.99 kg to make the math slightly easier.) *If the Sun starts out as pure hydrogen and converts entirely into helium via nuclear fusion, how long can it last?*

(Remember that $c = 3 \times 10^5\,\text{km/sec} = 3 \times 10^8\,\text{m/sec}$. Express your answer in years. Write down the numbers you are multiplying and dividing as well as your final answer.)

In practice, only the central 10% of the Sun gets hot enough for fusion, the Sun is initially 75% hydrogen instead of 100%, which (together with the difference between 0.99 and 0.993) makes the lifetime of the Sun about 15 times lower than this.

Part III: A Time Dilation Experiment

Each part of the question is worth 5 points. The background to this question will be discussed in next week's lectures.

(a) Suppose your watch is running at $f = 0.9 = 90\%$ of the rate it should be running. We could also describe this by saying that your watch is running slow by $1 - f = 0.1 = 10\%$.

You synchronize your watch with an accurate clock at noon. *When an hour has passed and the accurate clock says 1 pm, what time will your watch say?*

For future reference, note that in this example your watch ends up behind the true time by an amount

$$\Delta t = t_{\text{true}} - t_{\text{watch}} = t \times (1 - f),$$

where $t = 1\,\text{hr}$ is the amount of time passed and Δt (pronounced "Delta-t") is the time difference.

(b) The circumference of the earth is about 40,000 km, or 4×10^7 m. There are $24 \times 60 \times 60 = 86,400$ seconds in a day. *If you live on the equator, what is your speed v (caused by the earth's rotation) relative to a hypothetical (and very hot) observer who is at rest in the center of the earth?*

Give your answer in m/sec. Also give the ratio v/c of this speed to the speed of light.

(c) Since the earth is orbiting the sun, it's not obvious that you can treat the center of the earth as an "inertial frame," but for purposes of this problem you can do so because the earth is in free fall. A clock at the equator moves at speed v relative to a hypothetical clock at the center of the earth, so according to Einstein's theory of relativity it runs slow by an amount

$$1 - f = 1 - \sqrt{1 - \left(\frac{v}{c}\right)^2}.$$

Unless you have a clever calculator and know how to use it well, when you plug your value of v/c into this formula you will get the (incorrect) answer of zero. I will therefore give you a near-perfect approximation to use instead:

$$1 - f = \frac{1}{2}\left(\frac{v}{c}\right)^2.$$

(This approximation works when v/c is much smaller than one, as it is here.)

Consider an interval $t = 40$ *hours* (you'll see why below). *Over this time interval, what is the accumulated difference in time between a clock rotating with the earth at the equator and a clock at rest in the earth's center?*

You can use the same formula as above:

$$\Delta t = t_{\text{center}} - t_{\text{surface}} = t \times (1 - f).$$

Give your answer in seconds. (It will be much less than one second.)

(d) Unfortunately there are no clocks at the center of the earth (at least as far as we know), so you can't do this experiment. However, you can put a clock on an airplane. A typical commercial jetliner goes at about 1000 km/hr, which is 278 m/sec.

Consider a clock on an airplane that is flying east around the equator, in the same direction that the earth rotates. *What is the speed of the clock relative to the clock at the center of the earth?*

Give your answer in m/sec. Also give the ratio v/c of this speed to the speed of light.

(e) *How long does it take such a plane to go once around the earth at the equator?* Let this be the time interval t. *Applying the same reasoning as part (c) above, what is the accumulated difference between the flying clock and the hypothetical clock at the earth's center,* $\Delta t = t_{\text{center}} - t_{\text{airplane}}$*?*

(f) *Combining your answers to (c) and (e), what is the difference between the clock on the airplane and the clock on the surface? When the airplane gets back to where it started, is the time on its clock ahead or behind the time on the surface clock?*

(g) *If you have clocks that can measure time with an accuracy of* 10^{-8} *seconds (a.k.a. 10 nano-seconds), would you be able to measure this difference?*

(h) Read the Wikipedia article on the "Hafele-Keating experiment". *How is that experiment related to the calculation you did in parts (a)-(e)?* (Two or three sentences is sufficient.)

Extra credit (5 points): Suppose that the plane flies west around the equator of east. What is different?