Astronomy 2142, Assignment 3: White Dwarfs, Neutron Stars, and Black Holes

This assignment is due at 5 pm on Monday, March 31, a slight shift from the usual Friday-to-Friday timeline. The preferred submission is electronic, via Carmen, preferably PDF. You may also submit it on paper, either in class or at my office in 4019 McPherson Lab (if my door is closed, slide it under). It's your responsibility to write clearly enough that we can grade your answers. Write your name on the assignment!

Late assignments will be marked down 10 points if turned in before midnight on March 31, or 15 points if turned in before 5 pm on Thursday, April 3. No assignments will be accepted after that time.

You may consult with others in the class when you are working on the homework, but you should make a first attempt at everything on your own before talking to others, and you must write up your eventual answers independently.

You are welcome to come to my office hours for advice. This will almost certainly be helpful if you are finding the assignment difficult. Please spend some time working on the assignment before you come to office hours so that you know what your questions are. Office hours this week are slightly adjusted from those on the syllabus, so look below.

In-person office hours, 4019 McPherson Laboratory (4th floor, SW corner)
Thursday, 3/27, 11am-12:00pm
Virtual office hours, Zoom 827 776 2849, Passcode A2142
Friday, 3/28, 9:15am-10:15am

You can also ask me questions after class, and/or you can contact our TA, Wynne Turner (turner.1839@osu.edu) to set up a time to get help.

Part I: Questions on Reading

This week's short questions are connected to the reading we have done about stars and stellar mass black holes. Answer each of the numbered questions below. One or two sentences is sufficient. Each question is worth 5 points.

Chapter 4

The 1920s saw the first observational evidence for white dwarfs, stars whose mass is comparable the sun's but whose radius is comparable to the earth's. (White dwarfs are now understood to be the end state of stars like the sun, after they have exhausted their nuclear fuel, but this understanding took several more decades to develop.) R. H. Fowler proposed (correctly) that white dwarfs are supported against gravitational collapse by the pressure of *degenerate electrons*, electrons that are moving erratically at high speed because they are squeezed into a small volume. In 1930, while on a boat from India to England, *Subrahmanyan Chandrasekhar* showed that if the gravity of the white dwarf is so strong that the electrons are moving close to the speed of light, then the degenerate electrons would be less "springy," i.e., providing less resistance to compression.

1. From this deduction, what did Chandrasekhar conclude about white dwarfs?

2. Why is this conclusion relevant to the existence of black holes?

3. What was the reaction of Sir Arthur Eddington, the most eminent British astronomer of his day, to Chandrasekhar's suggestion?

Chapter 5

Fritz Zwicky and Walter Baade identified the existence of *supernovae*, enormously powerful stellar explosions. Zwicky also proposed the existence of *neutron stars*. A neutron star is a ball of neutrons with the density of an atomic nucleus, but held together by gravity rather than the nuclear force (which holds normal atomic nuclei together). A neutron star is roughly the mass of the sun (slightly more) but only about 10 km in radius.

4. According to Zwicky (who turned out to be correct), what is the source of energy that powers a supernova explosion?

5. What did *J. Robert Oppenheimer* and his student George Volkoff show about neutron stars, analogous to Chandrasekhar's finding for white dwarfs? (Hint: see pp. 191-192.) Why is their finding relevant to the existence of black holes?

Chapter 6

With particular attention to pp. 254-257:

6. What different points of view were conveyed by the terms "frozen star" (used mainly in the Soviet Union) and "collapsed star" (used mainly in the West)? Why were these terms unsatisfactory?

7. What is the term that *John Archibald Wheeler* coined for these objects? What ideas does this term convey?

(For the last question, you can rely on your own reaction to this term.)

Part II: Moving Fast

Each part of the question is worth 5 points, except (e) and (f) which are worth 10 points.

(a) In Section 3 of the course, we obtained the equation $M = \frac{v^2 r}{G}$, which relates the orbital speed v and orbital radius r for an object in a circular orbit to the mass that it is orbiting around. Combine this equation with the value I have given you for the Schwarzschild radius of the sun, $R_{\mathrm{Sch},\odot} = \frac{2GM_\odot}{c^2} = 3\,\mathrm{km}$, to show that the orbital speed divided by the speed of light is

$$\frac{v}{c} = \sqrt{\frac{1.5\,\mathrm{km}}{r}} \times \sqrt{\frac{M}{M_\odot}}.$$

(b) The earth is 300,000 times less massive than the sun. Using the result from (a), show that a satellite orbiting at a distance $r = 7000\,\mathrm{km}$ from the center of the earth (about 600 km above the earth's surface) will orbit the earth in about 5,500 seconds, or about 90 minutes. Remember that $c = 300,000\,\mathrm{km/s}$.

(c) A typical white dwarf star has a radius similar to the earth but a mass $M = 0.6M_\odot$. How long would it take a satellite to orbit such a star at a distance $r = 7000\,\mathrm{km}$?

(d) A typical neutron star has a mass $M = 1.4M_\odot$ but a radius of only 10 km. How long would it take a satellite to orbit such a neutron star at a distance $r = 7000\,\mathrm{km}$? How long would it take to orbit at a distance $r = 12\,\mathrm{km}$?

(e) Returning to the equations $M = \frac{v^2 r}{G}$ and $R_{\mathrm{Sch}} = \frac{2GM}{c^2}$, give a mathematical argument (a couple of lines of equations and some words to explain them) that light should be able to orbit a black hole in a circle at a distance $r = \frac{1}{2}R_{\mathrm{Sch}}$.

I told you (correctly) in class that light can orbit a black hole in a circle at a distance $r = \frac{3}{2} R_{\rm Sch}$, so three times larger than your calculation implies. Why do you think your calculation gives the wrong answer?

(f) The gravitational energy released when a stellar core implodes to form a neutron star is approximately $E_{\rm grav} = GM^2/R$, where M is the mass of the neutron star and R is its radius. In Assignment 2, we saw that the total energy that the sun will release over its entire lifetime as a normal ("main sequence") star is roughly $E_\odot = 0.001 M_\odot c^2$, since about 10% of the sun's mass will fuse from hydrogen to helium and each kg of hydrogen produces about 0.99 kg of helium.

For a neutron star mass $M = 1.4 M_\odot$ and radius $R = 10\,\mathrm{km}$, what is the ratio of the gravitational energy $E_{\rm grav}$ released by the implosion that forms the neutron star to the total energy $E_\odot$ released by the sun during its lifetime?

Hint: Use $2GM_\odot/c^2 = 3\,\mathrm{km}$.

(Note that the energy from the sun is produced over 10 billion years while the energy of the neutron star implosion is released in a few seconds!)

Part III: X-ray Iron Lines from a Black Hole Accretion Disk

Each of the multiple choice questions is worth 5 points.

For a non-spinning black hole that is being fed with gas from a companion, the inner edge of the accretion disk is at $R = 3R_{\rm Sch}$. The Newtonian calculation you did in Part II.(a) implies that gas at this distance should be orbiting at about 40% of the speed of light (try plugging in $r = 9\,\mathrm{km}$ and $M = 1M\odot$).

While this value, $v/c = 0.4$, isn't perfectly accurate, it is good enough for our purposes, and you should use it in your calculations below.

We previously wrote the Doppler shift formula in terms of wavelength, but for our current purpose it is better to write in terms of energy:

$$E_o = \frac{E_e}{\left(1 + \frac{v}{c}\right)} \ .$$

Here E_e is the energy of a photon emitted by an atom near the black hole, and E_o is the energy that we observe for that photon when we detect it far from the black hole. Remember that for atoms moving *away* from you v/c is positive (so energy is reduced, redshift) and for atoms moving *towards* you v/c is negative (so energy is increased, blueshift).

Highly ionized iron atoms emit X-ray photons with an energy $E_e = 6.4\,\mathrm{keV}$. (For our purposes, you just need to know that a keV is a unit of energy.) Suppose that we use an X-ray telescope to detect the iron emission from a black hole with an accretion disk.

(a) Considering *just* the effects of Doppler shifts, what should be the energies of the highest energy photons that we detect?

A. 12.8 keV

B. 10.67 keV

C. 7 keV

D. 6.4 keV

E. 4.6 keV

(b) Considering *just* the effects of Doppler shifts, what should be the energies of the lowest energy photons that we detect?

A. 10.67 keV

B. 7 keV

C. 6.4 keV

D. 6 keV

E. 4.6 keV

There is an additional effect we have to consider, namely gravitational redshift. For photons emitted at a distance R from a non-spinning black hole, we should multiply the energy we computed using the Doppler formula by another factor

$$f = \sqrt{1 - \frac{R_{\rm Sch}}{R}} \, .$$

For $R = 3R_{\rm Sch}$, this factor is $\sqrt{2/3} = 0.82$.

(c) Considering *both* the effects of Doppler shifts and gravitational redshift, what should be the energies of the highest energy photons that we detect?

A. 13.0 keV

B. 10.67 keV

C. 8.7 keV

D. 6.4 keV

E. 5.3 keV

(d) Considering *both* the effects of Doppler shifts and gravitational redshift, what should be the energies of the lowest energy photons that we detect?

A. 8.7 keV

B. 6.4 keV

C. 5.3 keV

D. 4.6 keV

E. 3.7 keV

(e) A plot of the distribution of photon energies from the accretion disk should most closely resemble which of the examples below?