Astronomy 2142, Assignment 4: Supermassive Black Holes and Gravitational Waves

This assignment is due at 5 pm on Friday, April 11. The preferred submission is electronic, via Carmen, preferably PDF. You may also submit it on paper, either in class or at my office in 4019 McPherson Lab (if my door is closed, slide it under). It's your responsibility to write clearly enough that we can grade your answers. Write your name on the assignment!

Late assignments will be marked down 10 points if turned in before midnight on April 11, or 15 points if turned in before 5 pm on Tuesday, April 15. No assignments will be accepted after that time.

You may consult with others in the class when you are working on the homework, but you should make a first attempt at everything on your own before talking to others, and you must write up your eventual answers independently.

You are welcome to come to my office hours for advice. This will almost certainly be helpful if you are finding the assignment difficult. Please spend some time working on the assignment before you come to office hours so that you know what your questions are. Office hours this week are slightly adjusted from those on the syllabus, so look below.

In-person office hours, 4019 McPherson Laboratory (4th floor, SW corner)
Thursday, 4/10, 11am-12:00pm
Virtual office hours, Zoom 827 776 2849, Passcode A2142
Friday, 4/11, 9:15am-10:15am

You can also ask me questions after class, and/or you can contact our TA, Wynne Turner (turner.1839@osu.edu) to set up a time to get help.

Part I: Short Questions

Question 1 is worth 10 points; questions 2-5 are worth 5 points. Questions 2-5 are based on chapters 9 and 10 of the book, as well as the discussion in lecture.

1. Highly ionized iron atoms emit X-ray photons with a specific energy $E = 6.4$ keV. (Here "keV" stands for "kilo-electron-volt," but for our purposes you don't really need to know what that means.) When we observe black holes with X-ray telescopes, we detect these photons coming from iron atoms, but *some* are observed to have energies higher than 6.4 keV and *most* are observed to have energies lower than 6.4 keV.

(a) Why do *some* of the observed photons have energies above 6.4 keV?

(b) Why do *most* of the observed photons have energies below 6.4 keV?

2. What was Karl Jansky's contribution to astronomy?

3. The radio source 3C273, found in radio surveys of the 1950s, appeared to coincide with a visible star whose image was unremarkable but whose spectrum showed highly unusual emission lines. In 1963, Maarten Schmidt recognized that these emission lines were in fact the normal emission lines of hydrogen gas, but redshifted to substantially longer wavelengths. From this realization, how did he conclude that 3C273 was in fact an enormously luminous object, a million times more luminous than any star?

(You should explain Schmidt's chain of reasoning in 2-3 sentences.)

4. What does LIGO uses laser interferometry for? (Your answer should be more specific than "to detect gravitational waves." What is measured by laser interferometry?)

5. Why were laser interferometers ultimately able to detect gravitational waves while bar detectors were not? A one-sentence answer is sufficient.

Hint: See pp. 377, 386, 387.

Part II: Luminosity and growth of a supermassive black hole

Part (e) is worth 10 points; the other parts are worth 5 points.

For this problem, you will need to use the formulas given in §10 of the lectures

$$L_{\rm Edd} = 3 \times 10^4 L_\odot \left(\frac{M}{M_\odot}\right)$$

for the Eddington luminosity limit and

$$L = 1.2 \times 10^{12} L_\odot \left(\frac{\dot{M}}{1 M_\odot\,{\rm yr}^{-1}}\right)$$

for the luminosity of a non-spinning black hole accreting gas through a thin accretion disk.

Consider a $1.2 \times 10^9 M_\odot$ black hole that lives at the center of a galaxy with 10^{12} stars.

(a) What is the maximum luminosity of the black hole accretion disk (i.e., the Eddington luminosity limit)?

For parts (b)-(d), assume that the black hole is radiating at 50% of this maximum luminosity.

(b) Suppose that the combined luminosity of the galaxy's stars is $10^{12} L_\odot$. What is the ratio of the luminosity of the accreting black hole to the combined luminosity of the stars?

(c) What is the accretion rate $\dot{M}$ onto the black hole, in $M_\odot\,{\rm yr}^{-1}$?

(d) In 10^8 years, how much mass would be added to the black hole, in $M_\odot$?

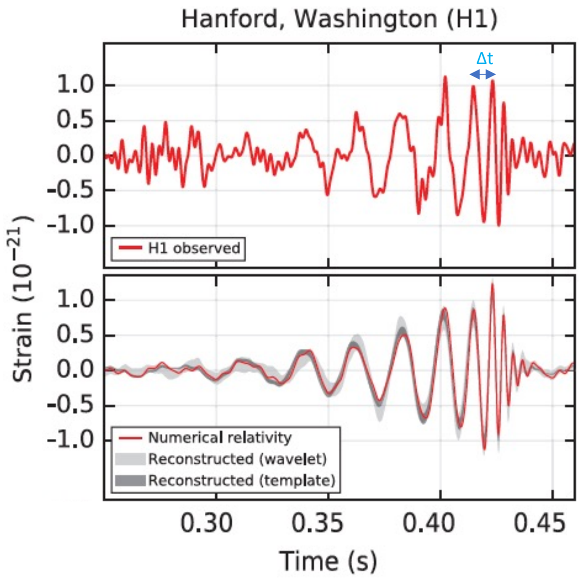
(e) The universe is about 14×10^9 years old. From your answers (especially to d), what can you conclude about whether or not this black hole has been shining at this luminosity for most of the age of the universe? Explain your reasoning in 1 or 2 sentences.

(f) You observe another galaxy with 10^{12} stars, but it does not have a luminous black hole. You suspect that the galaxy has a dormant black hole that is no longer being fed with gas. What measurements might demonstrate that the galaxy has a dormant black hole?

Part III: Gravitational waves from merging black holes

Part (e) is worth 10 points; other parts are worth 5 points.

The figure on the next page, taken from a paper published by the LIGO collaboration in 2016, shows the strain h as a function of time from the gravitational wave event GW150914, as measured by the interferometer in Hanford, Washington. The lower panel compares the observed signal to a theoretical prediction, but you should concentrate on the top panel that shows the observed signal itself.



(a) Measure the separation in time Δt between the last two strong peaks in this strain curve, as marked. Is this closest to 0.1 sec, 0.05 sec, 0.01 sec, or 0.003 sec?

(b) Using the value of Δt that you picked for part (a) and the formula

$$P \approx 0.0033 \left(\frac{M}{10 M_\odot} \right) \text{ sec} ,$$

what is the inferred mass M of the merging black holes, in solar masses?

(c) This formula is similar to the one given in class (see section 11 of the lecture notes), though the constant of proportionality has been changed from 0.001 to 0.0033 to make it more accurate. What is the physical argument behind this formula? In other words, why do we expect this to be the period of the gravitational waves emitted just before the black holes merge?

(d) Based on your answer to (b), what is the Schwarzschild radius R_{Sch} of the black holes (when they are still well separated before the merger)?

Hint: For the sun $R_{\text{Sch}} = 3\,\text{km}$.

(e) Using the formula (described on p. 365 of Thorne's book)

$$h = \frac{1}{5} \frac{R_{\text{Sch}}}{D} ,$$

together with the measured maximum strain $h = 10^{-21}$, what is the distance D to the merging black holes, in km?

(f) Using the conversion

$$1 \text{ light year} = (3 \times 10^5 \,\text{km}\,\text{s}^{-1}) \times (3 \times 10^7 \,\text{sec/yr}) = 9 \times 10^{12} \,\text{km} ,$$

what is the distance to the merging black holes in light years?

Extra credit (2 points): Why is the event called GW150914?