12. Inflation

Reading: Chapter 10, primarily §§10.1, 10.2, and 10.4.

The Horizon Problem

A decelerating universe has a *particle horizon*, a maximum distance over which any two points could have had causal contact with each other, using signals that travel no faster than light.

For example, in a flat matter-dominated universe with $a(t) \propto t^{2/3}$, a photon emitted at $t = 0$ has traveled a distance $d = 2c/H(t) = 3ct$ by time t.

When we observe two widely spaced patches on the CMB, we are observing two regions that were never in causal contact with each other.

More quantitatively, at $t = t_{\rm rec}$, the size of the horizon was about 0.4 Mpc (physical, not comoving), making its angular size on (today's) last scattering surface $\theta_{\rm hor} \approx 2°$.

The last scattering surface is thus divided into $\approx 20,000$ patches that were causally disconnected at $t = t_{\rm rec}$.

How do all of these patches know that they should be the same temperature to one part in 10^5?

The Flatness Problem

The curvature radius is at least $R_0 \sim cH_0^{-1}$, perhaps much larger.

The number of photons within the curvature radius is therefore at least

$$N_\gamma = \frac{4\pi}{3}\left(\frac{c}{H_0}\right)^3 n_\gamma \sim 10^{87},$$

where $n_\gamma = 413\,{\rm cm}^{-3}$.

The universe is thus *extremely* flat in the sense that the curvature radius contains a very large number of particles.

Huge dimensionless numbers like 10^{87} usually demand some kind of explanation.

Another face of the same puzzle appears if we consider the Friedmann equation

$$H^2 = \frac{8\pi G}{3c^2}\frac{\epsilon_{m,0}}{a^3} - \frac{kc^2}{a^2R_0^2}.$$

Even if the "energy" term and "curvature" term on the right hand side are similar in magnitude today, then at very early times (BBN, say, with $a \sim 10^{-9}$) the energy term was enormously bigger.

(Modern cosmological measurements show that the energy term is much larger than the curvature term today. In the radiation dominated regime, the energy term scales as a^{-4}, so the discrepancy is even worse.)

The textbook (§10.1) frames this in terms of the value of Ω: if Ω is approximately one today, it had to be extremely close to one at very early times.

Inflation in a nutshell

In the standard big bang model, the flatness and large scale uniformity of the universe are just accepted as initial conditions, not explained.

Inflation is an extension of the big bang model that attempts to give a causal explanation for the origin of flatness and homogeneity, much as the big bang model itself gives a causal explanation for the primordial helium abundance.

The basic idea and first concrete model of inflation was proposed in 1980, by Alan Guth, and quickly followed up by many others.

In the inflation scenario, the early universe went through an accelerating phase in which it was dominated by vacuum energy with an extremely high energy density.

During this phase, the universe expanded exponentially in time, with an e-folding timescale

$$t_{\rm exp} = t_H = \frac{1}{H} = \left(\frac{8\pi G\epsilon_{\rm vac}}{3c^2}\right)^{-1/2}.$$

If the universe expanded by at least a factor of e^{60} during inflation, then the entire volume of the presently observable universe was within one causally connected patch *before* inflation started, so causal processes could have established the homogeneity of the universe.

During inflation, $a(t)$ grew by a very large factor at constant ϵ, making the curvature radius very large and growing the energy term relative to the curvature term. After inflation, the universe is extremely flat.

Eventually, inflation ended, and the enormous energy that had been stored in $\epsilon_{\rm vac}$ was converted to photons and other particles, producing the very large number of particles within the curvature radius.

A natural prediction of inflation is that Ω_0 should be extremely close to 1.0.

Scalar field inflation

In most implementations of inflation, the accelerated expansion is driven by a *scalar field* ϕ that fills space.

This field is assumed to have a potential energy $V(\phi)$ (analogous to the potential energy $B^2/8\pi$ of a magnetic field with magnitude B).

For a scalar field, the total energy density and pressure are

$$\epsilon_\phi = \frac{1}{2}\frac{1}{\hbar c^3}\dot{\phi}^2 + V(\phi)$$
$$p_\phi = \frac{1}{2}\frac{1}{\hbar c^3}\dot{\phi}^2 - V(\phi).$$

ϕ has units of energy (e.g., GeV), while $V(\phi)$ has units of energy per unit volume.

If the field is changing slowly, so that the $\dot{\phi}^2$ terms are much smaller than $V(\phi)$, then we have $p_\phi \approx -\epsilon_\phi$ and thus a component that can produce exponential expansion if it dominates the total energy density.

Successful inflation thus requires a phase in which $V(\phi)$ dominates the energy and pressure budget for a sufficiently long time.

If $\dot{\phi}$ is small and $V(\phi)$ is sufficiently flat, then the universe goes through a phase of nearly exponential expansion with

$$a(t) \propto e^{Ht}$$

and Hubble parameter

$$H = \left(\frac{8\pi G\, V(\phi)}{3c^2}\right)^{1/2}.$$

The expansion is "nearly" exponential because ϕ, and hence $V(\phi)$, are changing slowly as the expansion goes on.

Depending on the form of $V(\phi)$ and the value of $\phi(t_i)$ at the start of inflation, the exponential expansion may last long enough (at least 60 e-folds) to solve the horizon problem and the flatness problem.

Many models associate inflation with the epoch at which grand unification breaks, expected to be at an energy scale $kT_{\rm GUT} \approx 10^{15}$ GeV, or with the Planck epoch when quantum gravity effects become important, at an energy scale $kT_{\rm Planck} \approx 10^{19}$ GeV.

Figure 11.3 of the book illustrates one commonly assumed form of the potential, which leads to inflation if ϕ starts near zero and eventually rolls to a minimum at some non-zero ϕ_0.

Another broad category of models, usually referred to as "chaotic inflation," has a potential something like $V(\phi) \propto \phi^4$, in which case the field must start at a *large* value, and inflation occurs while it rolls towards zero.

In either scenario, inflation ends when ϕ begins to oscillation about the minimum of $V(\phi)$, so that $\dot{\phi}$ terms dominate the energy density and pressure.

If there is some coupling between the field ϕ and other fields and particles (such as photons), then these oscillations will be damped and the energy will be dumped into these other fields and particles.

This is known as the *reheating* epoch.

In effect, gravitational potential energy associated with repulsive gravity has been used to make the universe much larger, but with the same energy density as before inflation. Sometimes described as "the ultimate free lunch."

After this epoch, we return to a normal, radiation-dominated, hot big bang model, but with a universe that is much larger and flatter and causally connected over much larger scales.

Inflation and flatness

Suppose that at the start of inflation the energy and curvature terms on the r.h.s. of the Friedman equation are comparable,

$$\frac{8\pi G}{3c^2}\epsilon \approx \frac{kc^2}{a^2 R_0^2}.$$

Suppose that during inflation, $a(t)$ grows by the minimum factor of e^{60} required to solve the horizon problem, and that ϵ stays constant.

At the end of inflation, the energy term is larger than the curvature term by a factor of $(e^{60})^2 = e^{120}$.

To examine the other manifestation of the flatness problem, suppose that inflation occurs at a time $t_{\rm GUT}$ and that the curvature radius at the start of inflation is approximately $R = aR_0 \approx ct_{\rm GUT}$.

Further suppose that at this time there is approximately one "particle" of energy $kT_{\rm GUT}$ in each volume $(ct_{\rm GUT})^3$.

The energy density is therefore $\epsilon = kT_{\rm GUT}/(ct_{\rm GUT})^3$.

When inflation ends, the curvature radius is larger by a factor of e^{60}.

However, the energy density is still the same, so there is still roughly one particle of energy $kT_{\rm GUT}$ per volume $(ct_{\rm GUT})^3$.

The number of particles within the curvature volume $\sim R^3$ is therefore at least $N \sim e^{180} \sim 10^{78}$.

Density fluctuations from inflation

Inflation "irons out" pre-existing inhomogeneities by stretching them out to enormous scales.

Soon after inflation was proposed, people realized that the scalar field driving inflation would experience quantum fluctuations in accordance with the Heisenberg uncertainty principle.

In an exponentially expanding universe, these fluctuations are stretched from microscopic scales to macroscopic scales, as different regions of the universe grow by slightly different factors and end up at slightly different densities.

Roughly speaking, at any time during inflation there are quantum fluctuations in the value of ϕ on scale cH^{-1} of magnitude $\delta\phi \sim H$.

These fluctuations cause inflation to end at slightly different times in different locations, with $\delta t \sim \delta\phi/\dot{\phi}$. This in turn leaves the universe with energy density fluctuations on this scale $\delta\epsilon/\epsilon \sim H\delta t$.

According to inflation, these quantum fluctuations from the very early universe are the source of density variations that produce anisotropy in the CMB and seed the gravitational growth of structure in the universe.

The inflation potential has to be "fine-tuned" to get CMB fluctuations as low as 10^{-5}. [It requires $V'(\phi)/V(\phi) \ll 1$.]

However, once this fine-tuning is done, the statistical properties of the fluctuations predicted by inflation are in extremely good agreement with the observed properties of CMB anisotropies and large scale structure in the universe.

Specifically, in agreement with observations, the predicted fluctuations are

- Gaussian (bell curve distribution of $\delta\epsilon$)
- Nearly scale invariant (the amplitude of fluctuations on scale ct at time t, $\delta_H(t) \sim 10^{-5}$, is nearly independent of t)
- Present equally in all forms of energy – specifically, the cold dark matter, baryons, and photons all fluctuate together. (Technically, these are referred to as "adiabatic" fluctuations.) This is crucial to getting the observed pattern of peaks and troughs in the CMB angular power spectrum, and to getting the observed polarization.

Assessment

Inflation is the best theory we have for explaining the flatness of the universe, the large scale uniformity of the universe, and the origin of inhomogeneity in the universe.

Inflation does not really change the “standard big bang” model, but it gives an explanation for things that in the standard model are just accepted as initial conditions.

CMB evidence for a flat universe confirms one of the key predictions of inflation: that Ω_0 should be equal to one within the limits of measurement.

CMB anisotropies also have the statistical properties predicted “generically” by inflation.

However, these properties (scale-invariant, adiabatic, Gaussian) are what one might have guessed even without a theory for their origin.

Today’s cosmic acceleration shows that accelerated expansion is possible, but the energy and timescale is very different from that needed for inflation ($\sim 10^{10}$ years vs. $\sim 10^{-32}$ seconds).

Inflation’s successes are impressive, but it relies on physics that we do not fully understand, and the evidence for inflation is not nearly as strong as the evidence for the hot early universe implied by BBN and the CMB.

A specific model of inflation that is rooted in particle physics discoveries *and* naturally predicts that the level of fluctuations in the CMB should be $\sim 10^{-5}$ would be very convincing, but we don’t have that yet.

The small departures from scale invariance confirm a “non-generic” prediction of inflation and give some insight into $V(\phi)$.

Detection of a contribution of gravity waves to CMB anisotropy would provide stronger evidence for inflation and much better insights into the energy scale at which it took place.

This gravitational wave signature could potentially be detected by CMB polarization measurements and is a major reason for the interest in CMB polarization experiments. However, it is not guaranteed to be present at a detectable level.

Current polarization measurements show that gravitational waves contribute less than about 5% of the large scale CMB anisotropy. This upper limit combined with the small departure from scale invariance already rules out some otherwise interesting versions of inflation. (See figure attached to web page.)

The “larger scale setting” for inflation — i.e., what preceded it and how the universe entered an inflationary state in the first place — is far from clear.