Astronomy 5682
Problem Set 4: Supernova Cosmology
Due Thursday, 3/9, 11:59 pm

Background Reading: Chapter 6. It is useful to read this chapter before starting the assignment.

You may turn in this assignment on paper, either in-class on Wednesday (3/8) or to my mailbox in 4055 McPherson on Thursday. Alternatively, you may submit it electronically via Carmen. Note that I have pushed the deadline to midnight on Thursday, since many of you couldn't quite make 4 pm last time. However, I think McPherson Lab gets locked around 6 pm, so if you're submitting on paper then plan accordingly.

IMPORTANT: You should turn in Graph 1 and Graph 2 (attached at the end) as part of your problem set solution. If you're submitting electronically, you can mark the graphs by hand and photograph them to include with your submission, or you can try to re-generate them with the same axes with whatever plotting software you like to use. If your marking by hand, I suggest doing it in pencil so that you can correct mistakes.

Personally, I find that plotting by hand gives me a more visceral feel for what is going on.

Background: Review of Magnitudes

The relation that is most often used to determine the distance to astronomical objects is

$$f = \frac{L}{4\pi d^2}, \tag{1}$$

the flux of energy received from a source is equal to the luminosity of the source divided by 4π times the square of the distance. The luminosity is the rate at which the source releases energy, and it is independent of the source's distance; it has units of energy/time (e.g., erg/s). The flux is the rate at which energy is received from the source per unit area; it has units of energy/time/area (e.g., erg/s/cm^2). Equation (1) is a straightforward consequence of energy conservation.

While fluxes and luminosities are relatively easy to understand, they are somewhat inconvenient to use in practice because astronomical objects span an enormous range of intrinsic and apparent brightness, leading to lots of numbers like 2.823×10^{17} and 6.81×10^{-11}. Instead, astronomers often give their measurements in terms of magnitudes, which are related to fluxes and luminosities by logarithms. The logarithms make formulas a bit more complicated, but because we are going to be using real astronomical data in this problem set, we don't have much choice but to bite the bullet and deal with magnitudes. This review is supposed to tell you all the things you need to know about magnitudes for the purposes of this problem set. All logarithms in this problem set are base-10. Also, in this problem set the letters m and M stand for apparent magnitude and absolute magnitude, respectively, not for mass.

The *apparent magnitude m* is a measure of apparent brightness related to the flux. The formula that relates flux to apparent magnitude is

$$m = -2.5 \log(f/f_0),$$

where f_0 is the flux of a star that would have apparent magnitude 0. As you can see from this equation, fainter stars (smaller f) have *larger* apparent magnitudes, and each reduction in flux by a factor of 100 corresponds to an increase of 5 magnitudes. If one star is 5 magnitudes fainter than another, then we receive 100 times less energy from it. If one star is 10 magnitudes fainter than another, then we receive $100 \times 100 = 10,000$ times less energy.

The *absolute magnitude M* is a measure of intrinsic brightness related to luminosity. The absolute magnitude of a star is defined to be the apparent magnitude that the star would have *if* it were at a distance of 10 parsecs. Note that while the apparent magnitude of a star (like the flux) depends on its distance, the absolute magnitude is an intrinsic property (like the luminosity) that does not depend on its distance.

By combining the definitions of apparent magnitude and absolute magnitude with the equation $f = L/(4\pi d^2)$, one obtains the equation that relates the apparent magnitude of an object to its absolute magnitude and distance:

$$m = M + 5 \log d_{\text{pc}} - 5,$$

where d_{pc} is the distance in parsecs. Since we will be dealing with distant galaxies and supernovae rather than stars in the Milky Way, it is more useful to use the equivalent equation

$$m = M + 5 \log d_{\text{Mpc}} + 25, \tag{2}$$

where d_{Mpc} is the distance in Mpc.

Equation (2) can be solved for d, yielding

$$d = 10^{0.2(m-M-25)} \text{ Mpc}. \tag{3}$$

Thus, if we know an object's absolute magnitude M and measure its apparent magnitude m, we can find its distance with equation (3), just as we could find the distance from equation (1) if we knew the object's luminosity and measured its flux.

The quantity $m - M$ is called the *distance modulus*. The distance modulus of a star or galaxy depends only on its distance; the larger the distance, the greater the distance modulus.

Equations (2) and (3) are the facts about apparent and absolute magnitudes that you need to use in this problem set. I should note that the flux of an object is usually measured through a filter, and that the apparent magnitude and absolute magnitude are therefore defined separately for each filter. In this problem set, we will be dealing entirely with data measured through a "visual" (V) filter, which selects out yellow light. The apparent and absolute magnitudes with such a filter would usually be written m_V and M_V, respectively, but to make our notation simpler we will just use m and M.

Problem 1: The Supernova Hubble Diagram (20 points)

A plot of apparent magnitude vs. redshift for distant astronomical objects is known as a "Hubble Diagram," since Hubble used such diagrams to demonstrate the expansion of the universe. The redshift z is related to the recession velocity v by the Doppler formula, which for $z \ll 1$ gives $v = cz$, where c is (as usual) the speed of light. We'll use cz in place of v, since the quantity cz is well defined even at redshifts large enough that the Doppler formula and Hubble's law begin to break down.

When a supernova explodes, its brightness increases rapidly, reaches a maximum, then declines over the course of a few weeks. There are at least two broad classes of supernovae, ones produced by core collapse in massive stars, and ones produced by exploding white dwarfs. The exact mechanism behind this latter class of supernovae, called Type Ia supernovae, is not well understood, but it appears that all Type Ia supernovae have approximately the same peak luminosity, i.e., that the absolute magnitude at maximum light M_{max} is nearly the same for all Type Ia supernovae (once they are corrected for the duration of the supernova, as I mentioned in class). For the rest of this problem set, we will use only Type Ia supernovae.

Table 1 below lists values of $\log cz$ and m_{max}, the *apparent* magnitude at maximum light, for a dozen supernovae discovered and observed in the early 1990s. The data are from a paper by Mario Hamuy and collaborators, published in 1996. These apparent magnitudes have already been corrected for the shape of the light curve, as discussed in class, to make the supernovae more nearly "standard candles."

Table 1

Supernova	$\log cz$	m_{max}
1990O	3.958	16.409
1990Y	4.066	17.3488
1991S	4.223	17.8124
1991U	3.992	16.3683
1992J	4.137	17.2548
1992P	3.897	16.2426
1992ag	3.891	16.2164
1992aq	4.481	19.0955
1992bc	3.774	15.4026
1992bk	4.240	17.7777
1992bo	3.736	15.4329
1993B	4.326	18.4524

(a) *Plot the positions* of these dozen supernovae on Graph 1.

(b) *Argue that* if (*i*) all supernovae have the same M_{max}, (*ii*) Hubble's law is correct, and (*iii*) peculiar velocities can be neglected, then all of these points should lie on a line of slope 5.0.

(c) I have included several lines of slope 5.0 on Graph 1. *Which of these lines* gives the best fit to the data? (Top, second from top, third from top, etc.)

This line can be described by the equation

$$m_{\mathrm{max}} = 5 \log cz + m_0. \tag{4}$$

What is m_0?

(It is useful to check your answer by showing that equation (4) fits two different points along the line.)

(d) *Why don't all the supernovae lie exactly on the line?*
(There may be several reasons. List those you can think of and assess which ones you think are probably the most important.)

Problem 2: Calibration of M_{max} and the Determination of H_0 (40 points)

If all went well, then Problem 1 convinced you that the supernovae in the above Table lie along a line of slope 5, and that the relation between their distances and redshifts is therefore well described by Hubble's law. Since Hubble's law states that $cz = H_0 d$, we can get $H_0 = (cz)/d$ if we can determine the distance to one of these supernovae.

Unfortunately, all of these supernovae are in galaxies that are too distant to have distances determined using Cepheids. However, the fact that they all lie close to a line of slope 5 implies that they all have nearly the same luminosity (same M_{max}), justifying the statement that I made in the second paragraph of Problem 1. Therefore, if we can determine M_{max} for a supernova in a nearby galaxy, we can assume that the supernovae in our diagram have the same M_{max}. In this Problem, we will use Cepheid variables to determine the distance to a galaxy where a well observed supernova exploded in 1990. (Actually, it exploded millions of years before, but the light reached us in 1990.) If you don't remember much about Cepheid variables, you might want to look them up in the textbook (§6.4) to remind yourself what they are and how they are used as distance indicators.

(a) Figure 1 plots the apparent visual magnitudes of Cepheid variables in the Large Magellanic Cloud (LMC) against the logarithms of their periods (measured in days). The data are taken from a paper by Barry Madore, who has done much of the work on Cepheids in the LMC.

Figure 1 contains a line that fits the average trend of the Cepheids fairly well. This line is described by the equation

$$m_C = a \log P + b_{\mathrm{LMC}}, \tag{5}$$

where the C stands for Cepheid. From Figure 1, *determine the values* of a and b_{LMC}.

(b) We really want the absolute magnitudes of the Cepheids, not the apparent magnitudes. To get this, we need a distance to the LMC. We'll adopt the value $d_{\mathrm{LMC}} = 50$ kpc $= 0.050$ Mpc, suggested by a variety of arguments that we won't go into here. Plugging this value into equation (2) yields

$$m = M + 5 \log(0.050) + 25 = M + 18.5 \tag{6}$$

for the relation between apparent and absolute magnitudes of stars in the LMC.

Apply equation (6) to your result from (a) to *determine the quantity* M_0 in the formula

$$M_C = a \log P + M_0 \tag{7}$$

that describes the relation between the period and *absolute* magnitude of a Cepheid variable. Note that M_0 is the absolute magnitude of a Cepheid with $\log P = 0$.

(c) Figure 2 plots apparent magnitude m_C vs. $\log P$ for Cepheid variables found in the relatively nearby galaxy NGC 4639, hereafter referred to as N4639. These data are taken from a 1997 paper by Abi Saha, Allan Sandage, and collaborators, who used Hubble Space Telescope to find and observe the Cepheids in this galaxy.

The line in Figure 2 is described by an equation

$$m_C = a \log P + b_{\text{N4639}}. \tag{8}$$

The slope a is the same as in equation (5), but the offset b is different because the distance to N4639 is greater than that of the LMC. I have put the line at the level that Saha and collaborators claim gives the best fit to their data points. Using this line, *determine the value* of b_{N4639}, just as you determined b_{LMC} in part (a).

(d) Combine the result of (c) with the result of (b) to *obtain the distance modulus $m - M$* for the galaxy N4639. (You may need to look back at the review of magnitudes to remind yourself of the definition of distance modulus.)

When you derive the distance modulus in this way, you are making an implicit assumption about the properties of Cepheids in the LMC and in N4639. *What is the assumption?*

(e) Supernova 1990N, the 14th supernova discovered in 1990, went off in the galaxy N4639. At its peak, it had an apparent visual magnitude

$$m_{\text{max}} = 12.61.$$

Combine this with the result of (d) to *determine the peak* **absolute** *magnitude M_{max}* of supernova 1990N.

(f) Now take your result from Problem 1c. Consider a hypothetical supernova with $cz = 10^4 \ \text{km s}^{-1}$, implying $\log cz = 4$. *What would its peak apparent magnitude m_{max} be* according to equation (4)?

Combined with your result from (e), *what would the distance modulus $m - M$ be* for such a supernova? *What distance (in Mpc) would then be implied* by equation (3)?

(g) *What is the implied value* of the Hubble constant H_0? Remember to give the units.

(h) *What are the main uncertainties in this result?*

Note: Your value may not agree exactly with the value I have quoted as a "best" estimate in class. However, if your value is wildly different (factor of two, say, or orders of magnitude), then you did something wrong.

Problem 3: Has Cosmic Expansion Been Slowing Down? (50 points)

You showed in Problem 1b that the apparent magnitudes and $\log cz$ of supernovae in galaxies obeying Hubble's law should lie on a straight line of slope 5. However, if we go to redshifts z that are not much smaller than 1, we must consider the effect of the changing rate of cosmic expansion.

For parts (a)-(d) of this problem, we will assume that the universe is matter dominated, with no cosmological constant. Thus, the value of Ω_0 is really just the value of $\Omega_{m,0}$, the matter density divided by the critical density.

Suppose that $\Omega_0 \approx 0$, so that the expansion *isn't* slowing down. Light received from a galaxy with a redshift $z = 0.5$, say, was emitted when the universe was smaller by a factor $(1 + z) = 1.5$. Expanding from that epoch to the present day took some t, and in that time the light traveled a distance $d = ct$.

Now suppose that $\Omega_0 = 1$. The universe was expanding faster in the past, so expanding from $1/1.5 = 2/3$ of its present size to its present size took less time than in the $\Omega_0 \approx 0$ case. Light therefore didn't travel as far.

(a) Based on this reasoning, *explain in words* how one could use the measured apparent magnitudes and redshifts of distant supernovae (with $z \sim 0.5$, say) to estimate the value of Ω_0. *Should high-redshift supernovae* appear fainter for $\Omega_0 = 1$ or for $\Omega_0 \approx 0$?

(b) Give a mathematically rigorous version of your answer to (a), using the formula for "luminosity distance" $d_L = S_k(r)(1 + z)$ together with the integral formula discussed in class for the comoving distance r (p. 2 of the §6 course notes). You don't need to solve this integral, but you should *describe how changing the value of Ω_0 influences its value* – i.e., *does changing Ω_0 from zero to one* make r bigger or smaller? The value of Ω_0 also affects the luminosity distance through curvature, which determines the form of $S_k(r)$. *Does this effect go in the same direction* as the effect on r, or does it go in the opposite direction?

(c) As discussed in class, the expected relation between m and z can be summarized for the luminosity distance, which is the distance d that belongs in equation (2). For $H_0 = 65 \ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, the formulas for a matter-dominated universe with $\Omega_0 = 1$ and $\Omega_0 = 0$ are:

$$d_{\mathrm{Mpc}} = 9200(1 + z - \sqrt{1 + z}), \qquad \Omega_0 = 1$$

$$d_{\mathrm{Mpc}} = 4600z\left(1 + \frac{z}{2}\right), \qquad \Omega_0 = 0.$$

(In part h below, you'll address whether using $H_0 = 65 \ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ vs. some other value makes a difference to the result of this problem.)

Using these formulas, equation (2), and your derived value of the supernova peak absolute magnitude $M_{\max}$ from Problem 2e, *plot smooth curves* showing the predicted *apparent* supernova magnitudes $m_{\max}$ vs. $\log cz$ on Graph 2. In order to make your life (much) easier, I provide in Table 2 a tabulation of z, $\log cz$, and values of d_{Mpc} and $5 \log d_{\mathrm{Mpc}}$ for both values of Ω_0, at a number of values of z. You can plot points for these values of z on Graph 2 using the Table, then connect these points with smooth curves. *Label which curve* is for $\Omega_0 = 1$ and which curve is for $\Omega_0 = 0$.

Table 2

z	$\log cz$	$(\Omega_0 = 1)$ d_{Mpc}	$(\Omega_0 = 0)$ d_{Mpc}	$(\Omega_0 = 1)$ $5 \log d_{\mathrm{Mpc}}$	$(\Omega_0 = 0)$ $5 \log d_{\mathrm{Mpc}}$
0.02	3.778	92	93	9.83	9.841
0.05	4.176	232	236	11.83	11.86
0.1	4.477	471	483	13.36	13.42
0.2	4.778	962	1012	14.92	15.03
0.3	4.954	1470	1587	15.84	16.00
0.4	5.079	1994	2208	16.50	16.72
0.6	5.255	3083	3588	17.44	17.77
0.8	5.380	4217	5152	18.12	18.56
1	5.477	5389	6900	18.66	19.19

(d) *Plot the points* from Table 1 (given back in Problem 1) on Graph 2.

(e) In the mid-1990s, two large collaborations began searching for and measuring the properties of high-redshift supernovae. Table 3 lists data on 10 high-redshift supernovae observed by one of these groups, taken from a 1998 paper by Adam Riess and (many) collaborators. *Plot these points* on Graph 2.
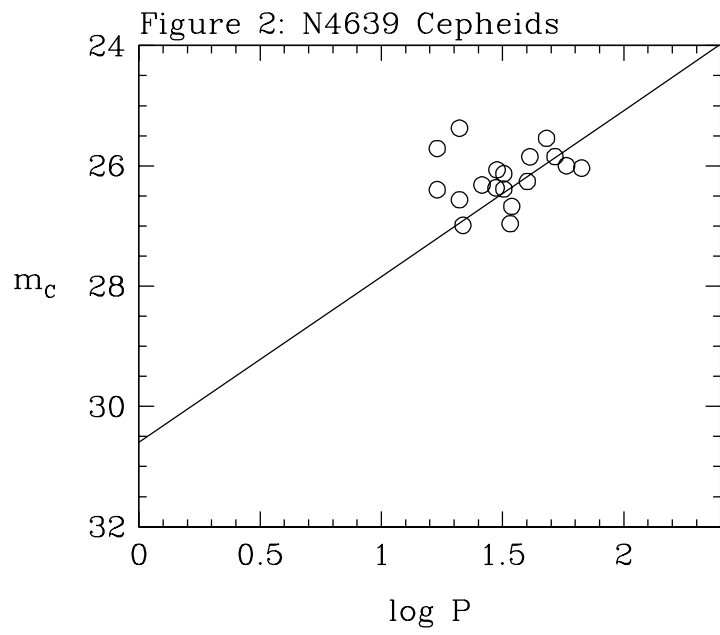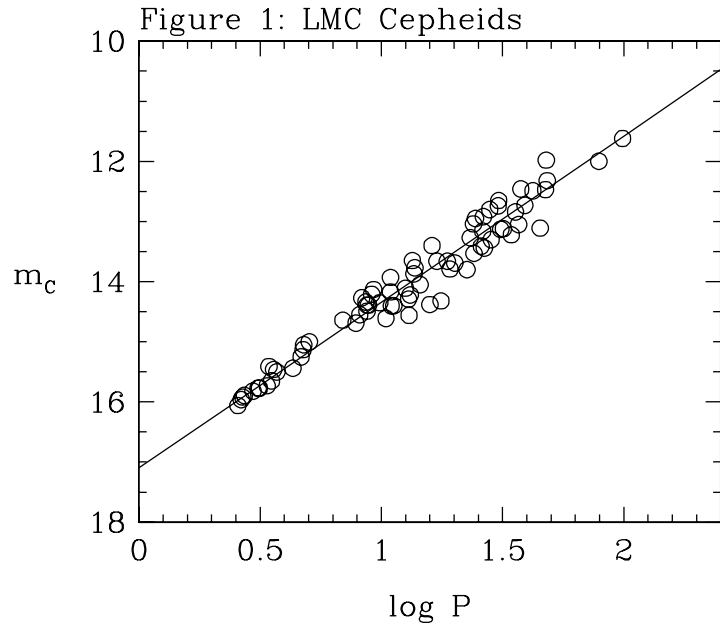
Table 3

Supernova	z	$\log cz$	$m_{\max}$
1995K	0.48	5.158	23.24
1996E	0.43	5.111	22.78
1996H	0.62	5.27	23.76
1996I	0.57	5.233	23.58
1996J	0.3	4.954	21.74
1996K	0.38	5.057	22.96
1996U	0.43	5.111	23.09
1997ce	0.44	5.121	23.01
1997cj	0.5	5.176	23.45
1997ck	0.97	5.464	25.05

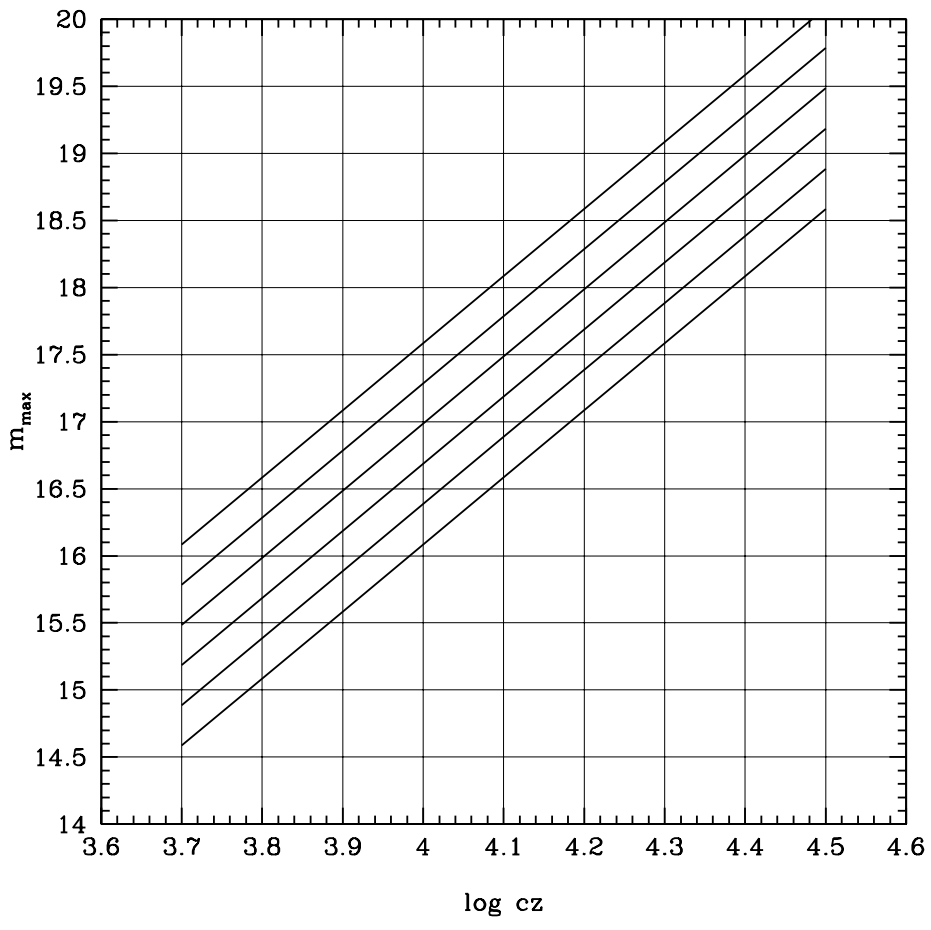(f) *What can you conclude* about the evolution of the cosmic expansion rate?
(Think carefully. The conclusion that you can draw is a major one.)

(g) *What are possible sources of error* in this result?

(h) Suppose that new observations provided a convincing measurement of the distance to the galaxy N4639, showing that its distance modulus was larger than the one that you derived in Problem 2d. *Would this change your conclusion* about the value of the Hubble constant in Problem 2? *Would it change your conclusion* in part (e) of this Problem about the evolution of the cosmic expansion rate? (Think carefully about what information in Graph 2 leads to the value of H_0 and what information in Graph 2 leads to conclusions about the evolution of the expansion rate.)

Figure 1: LMC Cepheids

Figure 2: N4639 Cepheids

8

GRAPH 1

GRAPH 2

m_{max}

log cz