Radiative Gas Dynamics

Problem Set 3

Due Tuesday, Feb. 6

Question 1

(a) Write down the equations that describe mass conservation and momentum conservation for *steady-state*, *spherically symmetric* accretion onto a compact object of mass M. Ignore viscosity and self-gravity of the gas. Argue that the quantity $\dot{M} = -4\pi r^2 \rho(r) u(r)$ is constant, and give a physical interpretation of $\dot{M}$. (The sign convention is that infall corresponds to negative u.)

(b) Now suppose that the object is accreting from a uniform medium whose density far from the accreting object (large r) is ρ_0. Assume an isothermal equation of state, $P = a^2\rho$ with constant sound speed a.

Show that

$$\frac{u^2}{r}\left(2 + \frac{\mathrm{d}\ln\rho}{\mathrm{d}\ln r}\right) = \frac{a^2}{r}\left(\frac{GM}{a^2 r} + \frac{\mathrm{d}\ln\rho}{\mathrm{d}\ln r}\right).$$

(c) Define the "Bondi radius" $r_B = GM/a^2$. Why is this a natural radius unit of the problem? What is its physical significance?

(d) At what radius (in units of r_B) is the infall velocity equal to the sound speed? If the density at this radius is $\lambda\rho_0$, what is the mass accretion rate in terms of λ, G, M, and a?

Question 2

For fully ionized, primordial composition gas, the dominant cooling process is free-free emission, with a radiative cooling rate

$$\Lambda \approx 2.5 \times 10^{-23} \left(\frac{T}{10^8\,\mathrm{K}}\right)^{1/2} \left(\frac{n_H}{1\,\mathrm{cm}^{-3}}\right)^2 \quad \mathrm{erg\ cm^{-3}\ s^{-1}}.$$

Consider a galaxy cluster that can be modeled as an isothermal sphere with core radius $r_c = 100$ kpc, and core density $n_H = 10^{-2}\,\mathrm{cm}^{-3}$ (treat the core as having constant density, which is an adequate approximation for our purposes). The cluster has a total (baryons + dark matter) mass of $4 \times 10^{14} M_\odot$ within a radius $R = 2$ Mpc.

(a) What is the cluster temperature in °K?

(b) What is the cooling time in the core of the cluster?

(c) If there is no heat input to balance this cooling, how do you expect the cluster to evolve on a timescale of ~ 5 Gyr?

(d) Suppose that a black hole in the cluster's central galaxy accretes mass at a rate $\dot{M}$ and injects kinetic energy with efficiency $\epsilon = 0.01\dot{M}c^2$. What $\dot{M}$ is required for this heating to balance the cooling rate of the core? How much mass would be added to the black hole over 5 Gyr?