XI. Cosmic Microwave Background (CMB) Anistropies

Huterer's Chapter 13 is very good.

Other excellent sources of information on CMB anisotropy are Wayne Hu's web site

`http://background.uchicago.edu`

and the Annual Reviews of Astronomy and Astrophysics article on *Cosmic Microwave Background Anisotropies*, by Hu & Dodelson 2002. Reading it is a substantial amount of work, but it covers all of the theoretical ground very well.

The 25-page Particle Data Group review by Scott & Smoot gives a capsule summary of the theory and up-to-date discussion of observations and their implications (find it through `https://pdg.lbl.gov`).

Brief History

Experimental searches for anisotropy followed soon after the discovery of the CMB.

Key early theory papers that captured much of the physics of CMB anisotropy are Sachs & Wolfe (1967), Silk (1968), Sunyaev & Zeldovich (1970), and Peebles & Yu (1970).

After decades of ground-based upper limits (plus balloon detection of the dipole in 1977, the Cosmic Background Explorer (COBE) satellite detected fluctuations at the 10^{-5} level on large angular scales, announced in 1992.

This was an epochal event in cosmology.

Theory papers by Hu and collaborators did a lot to make the theory of CMB anisotropy comprehensible to non-experts and to demonstrate the link between CMB observables and cosmological parameters.

This helped motivate the WMAP and Planck missions, which made CMB maps at much higher angular resolution and signal-to-noise ratio than COBE, enabling high-precision parameter constraints.

Current frontiers use ground-based telescopes and arrays with a focus on measuring CMB polarization, i.e., the polarization of CMB anisotropies.

Characterization of CMB Anisotropies

Suppose we have a map of the CMB temperature fluctuations $\Delta T(\theta, \phi)/T$ over the whole sky (where T in the numerator is the mean temperature).

Just as it is convenient to work with Fourier modes of 3-d fluctuations, it is convenient to decompose the CMB into spherical harmonics Y_{lm}:

$$\frac{\Delta T(\theta, \phi)}{T} = \sum_{l,m} a_{lm} Y_{lm}(\theta, \phi). \tag{11.1}$$

By definition $\langle a_{lm} \rangle = 0$. The mean squared expectation value of a_{lm} is called the angular power spectrum,

$$C_l \equiv \langle |a_{lm}^2| \rangle. \tag{11.2}$$

The typical temperature fluctuation for a CMB map smoothed over an angle θ is dominated by multipoles $l \approx \pi/\theta$ radians, with variance

$$\left\langle \left(\frac{\Delta T}{T} \right)^2 \right\rangle_\theta \approx l(l+1)C_l/2\pi|_{l\approx\pi/\theta}. \tag{11.3}$$

Even with perfect temperature measurements over the entire sky, there is statistical uncertainty in the power spectrum because there are a finite number of modes to measure.

This effect is known as *cosmic variance*, and it imposes a minimum statistical uncertainty

$$\frac{\sigma(C_l)}{C_l} = \sqrt{\frac{2}{2l+1}} \tag{11.4}$$

(Huterer eq. 13.79).

CMB Power Spectrum Measurements: First Look

The observed temperature fluctuation spectrum $l(l+1)C_l$:

- is flat from $l \sim 3 - 30$
- rises to a peak at $l \sim 220$
- shows a series of higher peaks of declining amplitude, with at least eight peaks clearly discernible
- second and third peak are similar in amplitude
- tails off slowly towards higher l
- has a low quadrupole ($l = 2$) relative to the low-l plateau, though the error bar is big (see, e.g., Fig. 29.2 of the PDG review, or many other sources)

Most of these features are well explained by an inflationary cold dark matter model with appropriate choice of the free parameters (which are numerous, but far fewer than the number of data points).

Large Angular Scales: The Sachs-Wolfe Effect

At large scales (low l), the dominant effects are intrinsic photon temperature fluctuations and gravitational redshifts.

Note that in an $\Omega_m = 1$, flat universe, the gravitational potential fluctuation $\Phi \sim -G\delta M/(Rc^2)$ is independent of time because $\delta M \propto a(t)$ and $R \propto a(t)$. (The division by c^2 makes Φ dimensionless.)

Adiabatic initial conditions

If initial conditions are adiabatic (isentropic), fluctuations are present equally in all species.

Regions of higher density and thus higher photon temperature are also CDM potential wells, which photons must climb out of.

The gravitational redshift effect wins: overdense regions are (slightly) colder, $\Delta T/T \sim \Phi/3$.

For scale-invariant fluctuations, the Sachs-Wolfe effect gives $l(l+1)C_l = \text{const.}$ (Huterer eq. 13.46).

Rough argument: $P_\Phi(k) \propto P(k)/k^4 \propto k^{-3}$ for scale-invariant ($n = 1$) fluctuation. The variance of Φ is $4\pi k^3 P_\Phi(k) = \text{const.}$.

Isocurvature initial conditions

For isocurvature fluctuations, regions that are overdense in CDM start out with lower photon temperature, so the two effects add instead of competing.

Overdense regions are much colder, $\Delta T/T \sim 2\Phi$, and the amplitude of fluctuations that corresponds to a given level of density contrast is six times higher.

Soon after the COBE discovery, isocurvature fluctuations were ruled out as the dominant initial conditions because they predict the wrong amplitude and wrong shape of the power spectrum.

Integrated Sachs-Wolfe

If $\Omega = \Omega_m = 1$, $\Phi \sim -G\delta M/R$ is independent of time because $\delta M \propto a(t)$ and $R \propto a(t)$.

If a photon falls into a potential well along the way to the observer, it must climb out of the same potential well, so there is no net redshift or blueshift.

But if radiation, curvature, or vacuum energy are important, then δM grows slower than $a(t)$, and the photon climbs out of a shallower potential well than it fell into.

The contribution to anisotropy from redshifts/blueshifts in time-varying gravitational potential wells is called the integrated Sachs-Wolfe (ISW) effect.

The "early" ISW effect occurs near recombination, where radiation is a significant contribution to the energy density, and it affects relatively small angular scales (a few times $ct_{\rm eq}/D_*$, where D_* is the angular diameter distance to the redshift of last scattering z_*).

The "late" ISW effect occurs at low redshift, if curvature or vacuum energy modify $a(t)$. It affects large angular scales, comparable to the present horizon; on smaller scales, a photon goes through many $+/-$ fluctuations while the potential decays.

Non-linear gravitational evolution also breaks $\delta M \propto a(t)$; this non-linear contribution to the ISW anisotropy is often called the "Rees-Sciama effect."

Intermediate Scales: The Acoustic Peaks

So far we have ignored pressure. But the photons are closely coupled to the baryons before recombination, and together they act like a fluid with a high sound speed $c_s = c/\sqrt{3(1+R)}$, with $R \equiv 3\rho_b/4\rho_\gamma$.

This fluid evolves in the potential wells provided by the (uncoupled) cold dark matter.

A key scale for characterizing the importance of pressure is the sound horizon, the comoving distance that a sound wave can propagate before the redshift of photon-baryon decoupling $z_* \approx 1090$.

The comoving size of the sound horizon is

$$r_* = \int_0^{t_*} \frac{c_s(t)dt}{a(t)} = \int_0^{a_*} \frac{c_s(a)da}{a^2 H(a)} \ . \tag{11.5}$$

The physical size at z_* is $a_* r_* \sim c_s t_*$.

Fluctuations on scales $\lambda < r_*$ oscillate in time.

Photons decouple from baryons at $z = z_*$, and we see a "snapshot" of the temperature distribution at that time.

A Fourier mode describes a pattern of $+/-$ fluctuations varying on a scale $\lambda \sim \pi/k$,

Since negative fluctuations do just the opposite of positive fluctuations in linear theory, we can get intuition for the evolution of Fourier modes by thinking about the evolution of a single, overdense, quasi-spherical perturbation of scale $\lambda \sim \pi/k$.

For fluctuation with $\lambda < c_s t_*$, there is time for a pressure gradient to build up, halt the initial collapse, and cause the perturbation to re-expand and overshoot the mean density.

The temperature fluctuation oscillates from positive to negative and back.

Decoupling catches oscillations at different phases, hence different amplitudes depending on scale.

Because modes of a given scale/frequency "start together" when they enter the horizon, they reach maximum amplitude — or zero amplitude — at the same time, regardless of their initial magnitude (think pendula of the same length that start from rest at different positions).

Oscillation of fluctuations in time with different frequencies leads to oscillation of typical $\Delta T/T$ amplitude with physical scale.

Contraction/expansion produces Doppler shifts, which are $90°$ out of phase with compression/rarefaction.

Therefore, the troughs between peaks don't fall to zero, but they are troughs because the Doppler shift effect is weaker than the actual energy density fluctuations.

Small Scale Damping

The last scattering surface has a finite width, $\Delta z \approx 80$.

Anisotropies are damped on scales smaller than this, since photons from positive and negative fluctuations are mixed together.

The damping scale is small, a few arc-minutes.

Projection

A given physical scale translates to an angular scale at the last scattering surface, $\theta \sim \lambda/D_*$, where D_* is the angular diameter distance at z_*.

Fourier modes have different orientations, so characteristic scales (e.g., of acoustic oscillations) are broader in angular space than in physical space.

The main effect in translating physical to angular scales is space geometry. *In an open universe, features appear at smaller angular scale and thus higher l.* (See Huterer Fig. 13.7.)

In a flat universe, D_* scales roughly as $\Omega_m^{-1/2}$, so higher Ω_Λ decreases angular scales for fixed physical scales.

However, the physical scale of the acoustic peaks is also approximately $\propto \Omega_m^{-1/2}$, so the angular scale of the peaks is only weakly sensitive to Ω_m for a flat universe.

Reionization Optical Depth

After reionization, free electrons in the intergalactic medium can re-scatter a small fraction of CMB photons.

This washes out fluctuations by mixing photons from different initial angular locations.

If the scattering optical depth is τ, a fraction $e^{-\tau}$ of photons are unscattered, and the amplitude of the power spectrum is multiplied by a factor $e^{-2\tau} < 1$.

This suppression does not occur on scales larger than (roughly) the size of the horizon at reionization, so the power spectrum is not suppressed on large angular scales (> a few degrees).

The optical depth can be estimated most precicely from the polarization of anisotropies. Final results from Planck imply $\tau \approx 0.06$, suppressing C_l by about 12%.

Secondary Anisotropies

Re-scattering at low redshift can generate new anisotropies as well as suppressing primary anisotropies.

The *thermal Sunyaev-Zeldovich effect* arises from scattering by hot gas (typically $kT_{\mathrm{gas}} >$ 1 keV, in galaxy clusters), which changes photon energies and distorts the blackbody spectrum. This produces a negative intensity fluctuation at low photon energy and a positive intensity fluctuation at high photon energy.

Doppler shifts at secondary scattering also induce anisotropy, often called the *kinetic Sunyaev-Zeldovich effect.* This requires ionization but not high temperature.

CMB Lensing

The CMB is gravitationally lensed by clustered matter at low redshift.

The lensing signal can be inferred in a few different ways, allowing statistical measurements and low resolution maps of the projected matter fluctuations.

The signal is dominated by clustering at $z \sim 0.5 - 5$, a fairly broad range.

CMB Polarization

Because scattering off of moving electrons produces polarization, CMB anisotropies are polarized.

The polarization fluctuations are about 10 times smaller than the temperature fluctuations ($\sim 10^{-6}$ instead of $\sim 10^{-5}$), so harder to measure.

Precise measurements of the polarization power spectrum increase the statistical information content of CMB anisotropies — crudely speaking, by a factor of two or three.

Planck has made pretty good measurements of the CMB polarization power spectrum, but ground-based experiments with bigger antennas are doing better except on the largest scales.

Anisotropies sourced by density fluctuations produce polarization patterns that are curl-free, referred to as *E-mode polarization.*

Roughly speaking, this is because displacements and velocities produced by gravitational potentials are curl-free (a.k.a. a "potential flow").

B-mode polarization with curl can be produced by gravitational lensing of E-mode polarization, which is one of the ways of isolating the effect of lensing on the CMB.

On large scales, B-mode polarization could be a sign of anisotropy induced by large scale gravitational waves (a.k.a. *tensor fluctuations*), which are predicted in inflationary models where the energy scale of inflation is high (not far below the Planck scale).

The search for this large-angle B-mode polarization is one of the major motivations for ambitious new CMB experiments (BICEP, Simons Array, CMB-S4, ...).

Accurate subtraction of contaminant signals (a.k.a. foregrounds), such as synchrotron and dust emission, is a major challenge for experiments seeking to measure these extremely small signals.

Putting It Together

CMB temperature and polarization anisotropies encode many physical effects:

Operating at $z = z_*$

- Primordial tensor fluctuations (gravitational waves)
- Photon energy density fluctuations $\Delta T/T$
- gravitational redshift of photons, $\Phi_*(\mathbf{x}_{\rm emit}) - \Phi_0(\mathbf{x}_{\rm obs})$
- Doppler shifts from fluid motion
- Acoustic oscillations
- Small scale damping

- Angular scale depends on $D_M(z_*)$, especially Ω_k

Operating at low z

- Time-variable gravitational potential
- Gravitational lensing
- Suppression of primary anisotropy by scattering after reionization
- Generation of secondary anisotropy by scattering after reionization: thermal and kinetic

These effects respond differently to initial conditions (e.g., from inflation) and to cosmological parameters (e.g., Ω_m, Ω_b, h, $\Omega_{\rm DE}$, Ω_k, Ω_ν, w, $N_{\rm eff}$, ...).

Although there are degeneracies, the effects are different in scale and in ΔT vs. polarization, so they can be largely separated.

CMB anisotropies are a powerful way to constrain cosmological parameters, especially in conjunction with low-z data (e.g., BAO, supernovae, galaxy $P(k)$) that have different degeneracies.

The CMB sets initial conditions from which one can predict low-z matter clustering and test theories of dark matter, dark energy, alternatives to GR.

Experimental frontiers

CMB as a probe of low redshift structure (lensing, Sunyaev-Zeldovich)

Large-scale B-mode polarization as a signature of primordial tensor fluctuations

Primordial non-Gaussianity as another probe of inflation physics

Astrophysics from CMB foregrounds