Problem Set 3: Primordial Nucleosynthesis
Due Wednesday, February 28

Chapter 7 of Huterer will be useful background for this problem set. Pay attention to Figure 7.3, especially for parts 7-9.

(1) The baryon-to-photon ratio

For a present-day microwave background temperature $T_{\mathrm{CMB}} = 2.73\,\mathrm{K}$, show that the ratio of baryons to photons is

$$\eta \equiv n_B/n_\gamma = 2.7 \times 10^{-8}\,\Omega_B h^2,$$

where $\Omega_B = \rho_B/\rho_c$ is the baryon density parameter with $\rho_c = 1.87 \times 10^{-29} h^2 \mathrm{g\,cm^{-3}}$ and $h \equiv H_0/(100\ \mathrm{km\,s^{-1}\,Mpc^{-1}})$. What is the baryon-to-photon ratio at a redshift $(1+z) = 10^5$?

It will help to remember that the number density of photons in a blackbody background of temperature T is

$$n_\gamma = \int_0^\infty \frac{1}{\pi^2 c^3} \frac{\omega^2 d\omega}{e^{\hbar\omega/kT} - 1} = \left(\frac{kT}{\hbar c}\right)^3 \frac{1}{\pi^2} \int_0^\infty \frac{x^2 dx}{e^x - 1} \approx 0.244 \left(\frac{kT}{\hbar c}\right)^3.$$

(2) The age-temperature relation

Using the Friedmann equation for a $k = 0$, radiation dominated universe, show that the age of the Universe at a time when the photon temperature is T is

$$t(T) = \frac{1}{2H} = 1.71 \left(\frac{1\,\mathrm{MeV}}{kT}\right)^2\ \mathrm{s}.$$

(Remember that the Boltzmann constant can be written $k = 8.617 \times 10^{-5}\ \mathrm{eV\,K^{-1}}$.)

The above equation would be correct if photons were the only relativistic species present. As discussed in Huterer chapter 5, the cosmic neutrino background has an energy density that is 68% of the photon energy density, changing the normalization of this equation by a factor $1.68^{-1/2}$ to

$$t(T) = \frac{1}{2H} = 1.32 \left(\frac{1\,\mathrm{MeV}}{kT}\right)^2\ \mathrm{s}. \tag{1}$$

Use this equation for your subsequent work.

(3) Neutron freeze-out

When the universe is sufficiently hot and dense, the ratio of neutrons to protons is maintained at the thermal equilibrium value

$$\frac{n}{p} = e^{-Q/kT}, \quad Q \equiv (m_n - m_p)c^2 = 1.2934\,\mathrm{MeV},$$

by reactions like $n + \nu \longrightarrow p + e^-$ that convert neutrons to protons or vice versa. However, in an expanding, cooling universe, the reaction rate is a rapidly decreasing function of time, and once the time for a particle to experience a conversion reaction gets longer than the age of the universe, conversion stops, and the n/p ratio "freezes out." An approximate expression for the conversion time per particle in the relevant temperature range is

$$t_c \approx 0.5 \left(\frac{1\,\mathrm{MeV}}{kT}\right)^5\ \mathrm{s}. \tag{2}$$

By setting this equal to the expansion timescale t from equation (1), derive a value for the freeze-out temperature T_F (expressed by kT_F in MeV) and the corresponding freeze-out value of the n/p ratio.

(4) Time of deuterium synthesis

All usable routes to the synthesis of heavier elements start with the synthesis of deuterium ($n+p \longrightarrow D+\gamma$). Deuterium is the most weakly bound nucleus, with a binding energy $B_D = 2.22\,\mathrm{MeV}$. Naively, one might expect that deuterium would form when kT falls to $\sim B_D$, since the typical photon would no longer be able to dissociate a deuterium nucleus. However, as shown in part (1), the number of photons per baryon is enormous, so rare photons on the high-energy tail of the Planck distribution may be abundant enough to dissociate deuterium even when kT is substantially smaller than B_D. A crude approximation that gives about the right answer is to say that deuterium is synthesized rapidly when $e^{-B_D/kT} \approx \eta$, where η is the baryon-to-photon ratio. Using this approximation and a baryon density $\Omega_B h^2 = 0.02$, deduce the temperature T_D and the time t_D at which deuterium synthesis occurs.

(5) Neutron abundance at deuterium synthesis

After the freeze-out described in (3), the neutron abundance drops slowly because of free neutron decay, which has an e-folding time $\tau_n \approx 880\,\mathrm{s}$ (half-life 610 s). What is the n/p ratio at the time of deuterium synthesis t_D?

(6) The helium mass fraction

At the density and temperature prevailing at t_D, the reactions that produce deuterium and the reactions that convert deuterium to ^{4}He are very fast, but almost no ^{4}He is processed to heavier elements. Thus, to a first approximation (one that is good enough to get the ^{4}He and H abundances but not the abundances of the other light elements), *all* of the neutrons existing at t_D are processed into helium. From the calculation in (5), what is the primordial helium mass fraction Y (the fraction of all baryonic mass that is in helium)? What is the hydrogen mass fraction X? Give results to two decimal places.

(7) The deuterium abundance and $\Omega_B h^2$

To a second approximation, a residual deuterium abundance "freezes out" when the timescale of reactions that process deuterium to heavier elements becomes longer than the age of the universe. For $\Omega_B h^2 = 0.02$, numerical calculations predict a deuterium-to-hydrogen ratio $D/H \approx 2\times10^{-4}$. If $\Omega_B h^2 = 0.1$, would the predicted deuterium abundance be higher or lower? Explain your answer.

It is believed that processing in stars can reduce the abundance of deuterium below the primordial value but cannot increase it. If an observation yields a secure value of $D/H = 4\times10^{-4}$ in some astronomical system, what is the implication for $\Omega_B h^2$?

(8) The helium abundance and $\Omega_B h^2$

Would raising $\Omega_B h^2$ to 0.1 raise or lower the predicted helium abundance? Explain your answer. (Hint: think about what happens to t_D.)

(9) The impact of N_ν

The equation for $t(T)$ that we have been using assumes that there are three species of neutrinos (e, μ, τ), each of them sufficiently low mass to be highly relativistic at the time of big bang nucleosynthesis. Suppose that an additional light neutrino species were discovered. Would this raise or lower the predicted helium abundance? Explain your answer.