Problem Set 6: DESI and Cosmological Parameters
Due Wednesday April 24

Because a lot piles up at the end of the semester, you should limit your work on this problem set to 6 hours and get as far as you can in that time.

The goal of this problem set is to draw some inferences from the recently (!) released DESI Year-1 BAO measurements using a simplified analysis. As preparation, you should read sections 1 and 2 of `https://arxiv.org/abs/2404.03002` and glance through the plots in the remainder of the paper.

The BAO technique measures distances relative to the sound horizon r_d, and the cosmological parameter constraints depend on what prior one takes on r_d. The paper uses a couple of different choices, but we will use a fixed $r_d = 147.09$ Mpc (comoving), which in the range of cosmologies considered here is constrained to better than 0.06% by the values of $\Omega_m h^2$ and $\Omega_b h^2$ inferred from Planck CMB anisotropy measurements.

This assignment builds on PS 2, where you derived an equation for the evolution of $H(z)/H_0$ and wrote a program to compute the comoving distance and angular diameter distance to redshift z for different cosmological parameters.

The quantities measured in BAO experiments are the comoving angular diameter distance $D_M(z)$, which is related to the physical angular diameter distance of PS 2 by $D_M(z) = D_A(z) \times (1+z)$, and the "Hubble distance" $D_H(z)$ which is just $D_H(z) = c/H(z)$. As explained in the paper, when the BAO S/N is low, either because the volume is small or the tracers are sparse, DESI instead measures the isotropic BAO scale

$$D_V(z) = [zD_H(z)D_M^2(z)]^{1/3} \ .$$

To make life (possibly) easier for you, I have split Table 1 of the paper into the tables `desi.tbl1` and `desi.tbl2` available on the web page, separating the five redshifts for which D_M and D_H are measured from the two for which D_V is measured.

For this assignment, you need to modify your program to allow dark energy with an equation of state parameter w, for which

$$\frac{\rho_{\rm DE}(z)}{\rho_{\rm DE,0}} = (1+z)^{3(1+w)} \ .$$

A cosmological constant corresponds to $w = -1$. You also need it to calculate the comoving angular diameter distance for a universe with non-zero space curvature if it doesn't already do so.

So that you don't spend too much of your time on program modification and debugging, I have provided `cosmodist2.py` on the web page, which is my program for doing this. I recommend at least looking at this program to see what I've done, and you are free to use this program or to borrow from it for your own. My program also includes the radiation contribution to the energy density, which is a negligible effect in the redshift range probed by DESI but has a noticeable (though small) impact on the distance to recombination $D(z = 1090)$. I don't promise that my program is bug-free, though I have tested it a fair amount in the course of preparing this assignment. I like to write standalone programs that take command-line arguments and print output that I plot separately, but you can pull things into jupyter notebooks if that is your preferred style.

1. Models vs. data

As a fiducial set of cosmological parameters we'll take $\Omega_m = 0.30$, $H_0 = 69$ km/s/Mpc, $\Omega_k = 0$, $w = -1$; the last two choices imply a flat universe and a cosmological constant.

Compute $D_M(z)$, $D_H(z)$, and $D_V(z)$ for four suites of models:

- Fiducial H_0, Ω_k, w and $\Omega_m = 0.28, 0.29, 0.30, 0.31, 0.32$
- Fiducial Ω_m, Ω_k, w and $H_0 = 67, 68, 69, 70, 71$
- Fiducial Ω_m, H_0, w and $\Omega_k = -0.1, -0.05, 0.0, 0.05, 0.1$
- Fiducial Ω_m, H_0, Ω_k and $w = -1.2, -1.1, -1.0, -0.9, -0.8$

Plot curves for these models and the DESI data points. I find it works best to plot D_M, D_H, and D_V on the same plot with different line styles to represent the three quantities and different colors to represent the parameter choices. The behavior is different enough that this is not impossibly confusing.

If you can plot all four suites on the same page, it is helpful for comparison.

2. A closer look

To better see the distinctions between models and the comparison to data, it is helpful to divide predictions and measurements by those of the fiducial model.

Remake the plots from Part 1, but in each case divide the model predictions and the data points by that of the fiducial model. To save you some file manipulation, I have created `desi.tbl1.ratio` and `desi.tbl2.ratio` which give the DESI data points and errors scaled to the fiducial model.

3. Assessment

Comment on your plots from 1 and 2, including discussion of degeneracies between parameters and how the measurements can break these degeneracies.

4. χ^2 plots

For this part, I recommend using my code `desi_chisq.py` so that you don't spend all of your time on tedious bits of the coding.

For each of the four model sequences, plot $\Delta\chi^2$ vs. the varying parameter (Ω_m, H_0, Ω_k, w), where $\Delta\chi^2$ is the difference in χ^2 relative to that of the fiducial model.

What is the difference between the quantities `chisq1` and `chisq2` computed in my code, and why is the second one really the more appropriate one to plot?

What value of $\Delta\chi^2$ corresponds to a model being 10 times less probable than the fiducial model (likelihood ratio of 0.1)?

5. A high-z constraint

The angular scale of the sound horizon at recombination, θ_*, is measured almost perfectly by CMB data, giving better than 0.06% precision on the ratio $D_M(z = 1090)/r_d$.

For each of the four model sequences, plot the ratio $D_M(z = 1090)$ relative to that of the fiducial model. (For convenience, `desi_chisq.py` outputs this quantity.)

Why is this ratio especially sensitive to Ω_k?

Comment on how including θ_* in the BAO analysis can sharpen the cosmological parameter constraints by breaking degeneracies.