Short Problems: Gravitational Redshift

This assignment is for your own edification. You don't need to turn it in.

1. Reproduce for yourself the $E = mc^2$ argument given in class, based on momentum conservation of a box that emits EM radiation as observed from a frame moving at speed v.

2. Consider a box of height l with a uniform gravitational field of downward acceleration g (e.g., $g = 9.8$ m/sec^2 on earth). A laser at the top shines photons of frequency $\nu_{\rm top}$ down to the bottom of the box. By applying the equivalence principle and the standard Doppler formula

$$\nu_{\rm obs} = \nu_{\rm emit} \times \left(1 + \frac{v}{c}\right) ,$$

show that the frequency of photons when they reach the bottom of the box is

$$\nu_{\rm bot} = \nu_{\rm top} \times \left(1 + \frac{gl}{c^2}\right) .$$

[Hint: How would the velocity of a uniformly accelerating box change over a light crossing time.]

3. Show that the "falling" photon gains energy

$$\Delta E = h\nu_{\rm top} \times \frac{gl}{c^2} .$$

Comparing this to the kinetic energy that would be gained by a ball of mass m dropping from the top of the box, what do you notice?