

### Short Problems: Cosmic Expansion History

For reference, here is the Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{8\pi G}{3}\rho + \frac{kc^2}{a^2 R_0^2} = 0.$$

What is the form of the time dependence of  $a(t)$  for the following cases?

1. An empty universe,  $\rho = 0$ .
2. A flat, matter-dominated universe,  $\rho \propto a^{-3}$ .
3. A flat,  $\Lambda$ -dominated universe,  $\rho = \rho_\Lambda = \text{constant}$ .
4. The diagrams below plot  $a(t)$ . Which diagram corresponds to each of the cases below?
  - Matter-dominated universe,  $k = +1$
  - Matter-dominated universe,  $k = 0$
  - Matter-dominated universe,  $k = -1$
  - Flat ( $k = 0$ ) universe with significant energy contributions from both matter and cosmological constant

*What is wrong with the two diagrams you didn't pick for any of the above?*

