

Short Problems: Cosmic Expansion History

For reference, here is the Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{8\pi G}{3}\rho + \frac{kc^2}{a^2 R_0^2} = 0.$$

What is the form of the time dependence of $a(t)$ for the following cases?

1. An empty universe, $\rho = 0$.
2. A flat, matter-dominated universe, $\rho \propto a^{-3}$.
3. A flat, Λ -dominated universe, $\rho = \rho_\Lambda = \text{constant}$.
4. The diagrams below plot $a(t)$. Which diagram corresponds to each of the cases below?
 - Matter-dominated universe, $k = +1$
 - Matter-dominated universe, $k = 0$
 - Matter-dominated universe, $k = -1$
 - Flat ($k = 0$) universe with significant energy contributions from both matter and cosmological constant

What is wrong with the two diagrams you didn't pick for any of the above?

