Short Problems: Mean Free Path Before Recombination

You do not need to turn this in.

1. Using the facts that the CMB has a density of $413\,\mathrm{cm}^{-3}$ today, that the photon-to-baryon ratio is $\eta = 6.1 \times 10^{-10}$, and that 6/7 of baryons are protons, argue that the mean density of electrons at redshift z is

$$n_e = 2.16 \times 10^{-7}(1+z)^3\,\mathrm{cm}^{-3} \ .$$

2. Assuming that all atoms are fully ionized at $z = 1200$ (not quite correct, but close), show that the mean free path of photons at $z = 1200$ is 4×10^{21} cm.

3. Argue that the Hubble parameter at $z = 1200$ is

$$H(z) \approx H_0 \left[\Omega_{m,0}(1+z)^3\right]^{1/2} \ .$$

(At $z = 1200$, what is the main source of inaccuracy in this formula?)

4. Recalling that $cH_0^{-1} = 4.28$ Gpc $= 1.32 \times 10^{28}$ cm (for $H_0 = 70\,\mathrm{km\ s^{-1}\ Mpc^{-1}}$), compute the ratio of the mean free path from part 2 to the Hubble distances $cH^{-1}(z)$ at $z = 1200$.

Was the universe at $z = 1200$ extremely opaque, pretty opaque, sort of transparent, or very transparent?